

# Author's Accepted Manuscript

Doing battle with short sellers: The conflicted role of blockholders in bear raids

Naveen Khanna, Richmond D. Mathews



[www.elsevier.com/locate/jfec](http://www.elsevier.com/locate/jfec)

PII: S0304-405X(12)00132-8  
DOI: <http://dx.doi.org/10.1016/j.jfineco.2012.06.006>  
Reference: FINEC2184

To appear in: *Journal of Financial Economics*

Received date: 12 July 2011  
Revised date: 2 November 2011  
Accepted date: 3 November 2011

Cite this article as: Naveen Khanna and Richmond D. Mathews, Doing battle with short sellers: The conflicted role of blockholders in bear raids, *Journal of Financial Economics*, <http://dx.doi.org/10.1016/j.jfineco.2012.06.006>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting galley proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Doing battle with short sellers: the conflicted role of blockholders in bear raids<sup>\*</sup>

Naveen Khanna<sup>a</sup>, Richmond D. Mathews<sup>b, \*</sup>

<sup>a</sup>*Eli Broad College of Business, Michigan State University, East Lansing, MI, 48824, USA* <sup>b</sup>*Robert H. Smith School of Business, University of Maryland, College Park, MD, 20742, USA*

---

## Abstract

If short sellers can destroy firm value by manipulating prices down, an informed blockholder has a powerful natural incentive to protect the value of his stake by trading against them. However, he also has a potentially conflicting incentive to use his information to generate trading profits. We show that a speculator can exploit this conflict and force the blockholder to buy a disproportionately large amount to prevent value destruction. This is costly for the blockholder because the trades must sometimes be executed at inflated prices. Given reasonable constraints on short sellers, a sufficiently large blockholder will have the incentive to absorb these losses and prevent a bear raid. However, conditions exist under which outside intervention may be warranted.

*Key words:* speculation, short selling, regulation, manipulation, bear raids

*JEL Classifications:* G18; G34

---

<sup>\*</sup> We thank an anonymous referee, Andres Almazan, Patrick Bolton, Brendan Daley, Jean-Etienne de Bettignies, Simon Gervais, Itay Goldstein, Alexander Guembel, Ron Kaniel, Adriano Rampini, David Robinson, S. Viswanathan, Jan Zabojsnik, and seminar/conference participants at Duke University, the Tuck School of Business at Dartmouth College, the Queen's School of Business, the University of Houston, Michigan State University, the University of Minnesota, the University of Illinois, the University of Oregon, the University of Maryland, Rice University, Texas A&M, the University of Texas at Austin, the 20th Annual Conference on Financial Economics and Accounting, the Fifth Annual FIRS Finance Conference, and the Third Annual Conference of the Paul Woolley Centre at LSE for helpful comments and discussions. All errors are our own.

<sup>\*</sup> Corresponding author. Tel.: + 1 301 405 4113; fax: + 1 301 405 0359.

*E-mail address:* rmathews@rhsmith.umd.edu (Richmond D. Mathews).

## 1 Introduction

Despite academic arguments that short selling improves market efficiency, many believe it is harmful and should be constrained. Their argument is often predicated on the belief that excessive stock price declines due to targeted short selling can make existing (or potential) creditors or other counterparties lose confidence in a firm and stop dealing with it. Thus, the damage is caused not so much by the initial drop in stock price, but through its feedback effect on the real decisions of the firm's counterparties, since that not only amplifies the firm's price drop but also makes it more permanent.<sup>1</sup>

Consistent with this reasoning, George Soros (2009) blames the demise of Lehman Brothers and American International Group on self-validating bear raids involving aggressive short selling of the firms' common stock. He claims that "the mispricing of financial instruments can affect the fundamentals that market prices are supposed to reflect." The Securities and Exchange Commission's defense of its 2008 short sales ban emphasized this same mechanism: "financial institutions are particularly vulnerable to [price declines due to short selling]...because they depend on the confidence of their trading counterparties in the conduct of their core business." Similar arguments form the basis for recent bans on short selling of bank stocks by some European Union countries. Exploitation of this feedback from prices to fundamentals has also been blamed for destructive price declines of many non-financial companies such as Overstock.com, an internet retailer, and Sedona Corp., a software company.<sup>2</sup>

---

<sup>1</sup> Several recent papers consider how this type of reverse causality may give rise to manipulation, including Khanna and Sonti (2004), Attari, Banerjee, and Noe (2006), and Goldstein and Guembel (2008). See pp. 4-6 for a complete discussion.

<sup>2</sup> In court filings, Overstock.com blames naked short sellers for "wrongfully, knowingly and intentionally act[ing] to interfere with and destroy or harm Overstock's existing and/or prospective business relationships" (Overstock.com, 2007). Similarly, Drummond (2006) quotes Sedona CEO David Vey as saying that stock declines due to naked short selling "make it difficult to attract new investors and capital and leaves potential customers wary."

In this paper we argue that this concern over manipulative short selling has overlooked the potentially critical role of blockholders who maintain long positions in the firms' stock. If there is reason to believe that short sellers may induce bad decisions by manipulating prices down, a blockholder has a powerful natural incentive to prevent such manipulation by buying enough shares to keep prices high. Thus, private markets should be able to handle value-destroying attempts by speculators without outside help.

We investigate this hypothesis with a model that includes a firm with an informed blockholder, a counterparty with a risk-averse<sup>3</sup> decision maker, and an uninformed speculator. The risk-neutral blockholder receives a signal about the value of a relationship between the firm and the counterparty, and can credibly convey his information to the counterparty's decision maker only through trading. The decision maker decides whether to accept the relationship based on the observed price. This gives the blockholder two objectives for his trading strategy—to ensure an efficient decision, and/or to use his information to maximize trading profits.

Since the decision maker is risk averse, inducing an efficient decision sometimes requires pushing prices to artificially high levels. In particular, when the blockholder's signal indicates a relationship of medium expected profitability he would like to have the relationship accepted, but the risk-averse decision maker may choose to reject. To prevent such a rejection the blockholder has an incentive to trade in a manner that suggests his signal is stronger. He can accomplish this in equilibrium if his trade after a medium signal is "pooled" with his trade after a high signal, so that the decision maker cannot distinguish between the signals.

However, this results in a conflict between the blockholder's two objectives. For the pooling strategy to be supportable in equilibrium, the blockholder must not have the incentive to deviate (or "separate") by trading a larger quantity after a high signal to increase trading profits, or

---

<sup>3</sup> An agency problem such as a private benefit or private cost for the decision maker would also suffice in place of risk aversion.

by trading a smaller quantity after a medium signal to avoid trading losses. Preventing the former requires a large enough pooled trade size so that equilibrium trading profits exceed those attainable with a larger trade. Preventing the latter requires a large enough block size to overcome the blockholder's reluctance to incur trading losses. Thus, if the blockholding is insufficient, the relationship may be rejected after a medium signal even though it is positive-NPV in expectation, causing an efficiency loss.

We next show that in this setting even an uninformed speculator can sometimes implement a profitable trading strategy by exacerbating the blockholder's internal conflict. The strategy involves arriving with a hidden initial position, and then (optimally) trading in the direction of that position. Since the main tension in the model is whether certain positive-NPV relationships will be lost, when the speculator arrives short, she wants to force prices down to induce rejection of these relationships ("raid" them), and thus sells. When she arrives long, she instead wants to raise prices to help ensure their acceptance, and thus buys.

This strategy makes it much harder for the blockholder to achieve efficiency because the speculator's actions make both of the deviations described above easier to hide and more profitable. This increases the size of the pooled trade needed to prevent separation with a high signal, while also increasing the size of the block needed to prevent separation with a medium signal. These two reinforcing effects lead to a disproportionate increase in the block size needed to ensure efficiency relative to the size of the speculator's trades. The identification of this friction between pooling to ensure efficiency and separating to maximize trading profits, as well as the speculator's ability to exacerbate this friction and cause a disproportionate increase in the block size needed, are some of the major contributions of our paper.

We assume the ability to short sell is exogenously constrained, so the increase in required block size due to the speculator's presence is measured *relative* to these constraints. Thus, given reasonable constraints, our analysis implies that blockholders should prevent value destruction

much of the time. However, outside intervention may be justified if short selling constraints become unexpectedly relaxed or in times of crisis when counterparties are likely more risk averse and blockholders less able to counter.<sup>4</sup> Importantly, though, we also find that the block size needed to ensure efficiency decreases in the extent of potential damage from a bear raid, both because it is more important to do so, and, surprisingly, because it is less costly to do so. The latter occurs because the increase in potential damage corresponds to an increase in expected value given a medium signal, which decreases the spread between that value and the pooled price and consequently reduces the required trading losses. Thus, the market has some ability to self-correct against the more excessive abuses of short selling.

We also show that bear raids are more likely to succeed in destroying value when blockholders' information is more precise, and when there are multiple informed blockholders who are unable to coordinate and do not individually hold large enough stakes. Both results are somewhat counterintuitive. The former occurs because extra precision increases potential trading profits, causing the blockholder to focus more on trading profits than on efficiency. The latter occurs because each independent blockholder faces the same trading incentives as does a single blockholder, and must trade just as aggressively at undesirable prices to ensure efficiency. Thus, each informed blockholder must hold a stake as large as that required for a single blockholder, and splitting up a block will leave each individual blockholder with insufficient stakes to prevent bear raids.

Our paper is directly related to Goldstein and Guembel (2008), who similarly model a short seller manipulating prices to induce bad decisions. However, unlike our paper, they do not consider the actions of other parties who may have an incentive to work against such a speculator. Furthermore, they require that the speculator sometimes be informed about firm fundamentals, while we do not. In our setting, it is the blockholder who is informed, and the decision maker

---

<sup>4</sup> In such cases, potential remedies lie also in incentivizing blockholders to increase their positions.

tries to infer the blockholder's information from prices that depend on his trade. Thus, decisions are sensitive to prices, which enables the speculator to affect decisions despite being uninformed. In addition, since the speculator starts with a non-zero position, she has an incentive to induce decisions that improve the value of her position even if she expects a loss from trading against the informed blockholder.

The existing literature on blockholders in corporate governance has established that there may be too little intervention when blockholdings are small because the costs are borne individually while the benefits are shared widely. While our paper fits within this general framework, we believe we are the first to consider how a blockholder's conflicted objectives toward the use of his private information can lead to profitable opportunities for manipulation by a strategic speculator. Furthermore, in our setting blockholders govern exclusively through trading, and the cost of governance is endogenously determined through the extent of trading losses or foregone trading profits. This unique interaction of the costs and benefits of governance inside the trading mechanism leads to the novel result of a disproportionate increase in the block size necessary for efficiency when the speculator is active.

The channel we identify for blockholder governance differs in important ways from those in the existing literature. The traditional channel is direct intervention in firm decision making, i.e., "voice" (see Shleifer and Vishny, 1986; Burkart, Gromb, and Panunzi, 1997; Bolton and von Thadden, 1998; Maug, 1998; Kahn and Winton, 1998; Faure-Grimaud and Gromb, 2004). This assumes the blockholder holds the necessary control rights to intervene directly. In our setting, the blockholder does not need control rights since he governs through his trades. Some other papers suggest that blockholders improve decisions by directly providing information to decision makers (see Cohn and Rajan, 2012; Edmans, 2011). Unlike our paper, this requires that the information be credibly communicable.

Others show that blockholders can, as here, govern through trading. Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011) show that the threat of informed selling, or “exit,” can improve stock price informativeness and encourage correct actions by managers. These papers rely on blockholders being more informed than other traders, which is shown by Edmans (2009) to arise from short selling restrictions. Thus, the blockholder’s significance would diminish with a relaxation of short sale constraints. In our setting, however, the blockholder’s significance is *greater* when short selling constraints are relaxed, since this increases the manipulation problem the blockholder tries to counter. Furthermore, in the “exit” papers blockholders trade only to maximize trading profits and the governance effect is a by-product, while here they explicitly trade off the two objectives.

Governance via trading is also studied by Khanna and Sonti (2004), who show that informed blockholders may manipulate prices upwards to influence managers to accept good projects and increase firm value; and by Attari, Banerjee, and Noe (2006), who show that institutional investors may strategically “dump” shares to induce activists to buy and then intervene directly in the firm’s management. As is true in our setting, the blockholders in these papers are motivated both by trading profits and by the desire to protect the value of their existing positions. However, neither paper considers the presence of a speculator who may actively disrupt the flow of information.

Our paper is also related to earlier analyses of the feedback effect from market prices to fundamentals like in Leland (1992), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Subrahmanyam and Titman (2001), and Ozdenoren and Yuan (2008).<sup>5</sup> In these papers, like here, price levels can induce firm or counterparty decisions that affect fundamental values. How-

---

<sup>5</sup> Also see, e.g., Durnev, Morck, and Yeung (2004), Luo (2005), Sunder (2004), Bakke and Whited (2010), Chen, Goldstein, and Jiang (2007), and Edmans, Goldstein, and Jiang (2011) for empirical evidence.



ever, these papers do not consider the possibility that market participants may manipulate prices to achieve desired outcomes.

A number of papers in the academic literature find that restrictions on short sellers tend to degrade market quality, and sometimes cause firms to be overvalued (see, e.g., Boehmer, Jones, and Zhang, 2009; Jones and Lamont, 2002; and Asquith and Meulbroek, 1996).<sup>6</sup> The latter finding is consistent with models of differences in beliefs, such as Miller (1977), but are at variance with Diamond and Verrecchia (1987), which argues that prices should be unbiased since markets will adjust for the truncated bad news. Duffie, Garleanu, and Pedersen (2002) suggest that overpricing may simply reflect the presence of lending fees.

Finally, our analysis is related to the general literature on manipulation. For example, Bagnoli and Lipman (1996) and Vila (1989) both study manipulation involving direct actions such as a takeover bid. Manipulation based on price pressure or information alone has also been studied widely, such as by Jarrow (1992), Allen and Gale (1992), and Chakraborty and Yilmaz (2004).

The paper proceeds as follows. The base model is specified in Section 2. The equilibria of the base model are characterized in Section 3. In Section 4 we endogenize the speculator's initial position. In Section 5 we consider an extension with multiple blockholders. Comparative statics and empirical implications are discussed in Section 6. Section 7 concludes. All proofs can be found in the Appendix.

## 2 The base model

We consider an economy with a single firm that has many indivisible equity shares outstanding.

A decision maker ( $D$ ) must decide whether to accept or reject a (new or existing) counterparty

---

<sup>6</sup> On the other hand, Kaplan, Moskowitz, and Sensoy (2010) study an exogenous shock to the supply of lendable shares for a random group of firms and find that there is very little effect on pricing or market quality.

relationship with the firm. Firm value is \$1 per share if  $D$  rejects the relationship. If  $D$  accepts,  $d \in (0, 1)$  per share is added to firm value if the future state of nature,  $\Theta \in \{B, G\}$ , is good ( $\Theta = G$ ), while  $d - \epsilon$  per share, where  $\epsilon \in (0, d)$ , is subtracted from firm value if the state of nature is bad ( $\Theta = B$ ). The ex ante probability of  $\Theta = G$  is  $\frac{1}{2}$ . This framework easily extends to situations where rejection of a counterparty relationship causes the firm to fail. The interpretation then is that the value upon failure is \$1 per share, and acceptance and hence continuation is efficient in the good state but not in the bad state.

There are (potentially) two strategic traders in the model: a risk-neutral, long-term investor,  $I$ , who possesses private information about the future state, and a risk-neutral speculator,  $S$ , who is uninformed about the state.  $I$  enters the game with a long position in the stock equal to  $i \geq 1$ , which is consistent with the empirical regulatory that firms often have one or more long-term blockholders (see Holderness, 2009). With regard to  $S$ 's presence we consider two specific cases, one with the speculator (the “active speculator” case) and another without (the “no speculator” case). In each case the speculator’s presence (or lack thereof) is common knowledge. Looking at the two cases allows us to compare regimes with and without speculation to determine the efficiency impact of the speculator’s actions. The no speculator case could correspond, for example, to a regulatory regime in which short selling is prohibited. In the active speculator case,  $S$  arrives with a privately known exogenous position that is long or short  $s \geq 1$  shares with equal probability. We later endogenize the initial position of the speculator by adding an earlier trading round, and verify that the speculator’s overall strategy can be profitable.<sup>7</sup>

<sup>7</sup> However, the model with an exogenous position for  $S$  remains useful in that it captures important scenarios where a speculator holds an *effective* position in a firm without ever having traded in its stock. For example, the speculator may hold a position in the securities of a competitor or potential acquirer (generally an effective short interest), or a supplier or customer (generally an effective long interest). Kalay and Pant (2008) discuss many such possible “correlated” long and short positions that occur without direct trading in the firm’s shares.

In the base model there is a single trading round. Before trading takes place,  $I$  receives a signal,  $\theta \in \{L, M, H\}$ , about the future state of nature, where H is high, M is medium, and L is low. The probability structure of the signals is such that

- $Pr[\theta = H|\Theta = G] = Pr[\theta = L|\Theta = B] = \lambda$ ,
- $Pr[\theta = H|\Theta = B] = Pr[\theta = L|\Theta = G] = \frac{1}{2} - \lambda$ , and
- $Pr[\theta = M] = \frac{1}{2}$

We assume  $\lambda \in (\frac{1}{4}, \frac{1}{2})$  so that the H and L signals are informative in the correct direction (i.e., an H signal implies a higher probability of the good state). With an M signal,  $I$  is effectively uninformed. No other agents receive any signals regarding the state, and the only way for  $I$  to communicate his information to  $D$  is through his trading strategy.<sup>8</sup> While our assumption that  $I$  receives a private signal but  $D$  does not is standard in the feedback literature, all we require is that  $I$  have access to some information that is incremental to  $D$ 's.

During the trading round, with probability  $\frac{1}{2}$  a noise trader places a market order to buy one share and with probability  $\frac{1}{2}$  it places an order to sell one share.  $I$  can place a market order for any integer quantity.  $S$  can place a market order to buy or sell one share, or can choose not to trade. This limit on the speculator's trades captures real-life constraints on short selling.<sup>9</sup> Limiting the speculator's trades endogenously determines how much  $I$  will choose to trade in equilibrium, implying that the interpretation of our results should always be *relative*. So if over

<sup>8</sup> It is natural to ask whether direct communication is a superior solution. However, in our model the blockholder does not want to fully reveal his information either to the market or to  $D$  because of both his incentive to make trading profits and his incentive to get the right decision made. Furthermore, since the decision maker resides outside the firm, and the single decision maker we model may actually represent numerous such agents across different counterparties, a direct communication mechanism may be infeasible in practice. Our assumption is also consistent with the existing feedback literature, which assumes that direct communication is not possible.

<sup>9</sup> Note that it is easy to show that  $S$ 's willingness to *buy* additional shares is endogenously limited by the extent of her long position. However, the short sale limit is a binding one—a short speculator would often wish to sell additional shares if she could. A number of empirical papers show that short selling is more expensive and more constrained than taking long positions (see, e.g., D'Avolio, 2002; and Geczy, Musto, and Reed, 2002).

some range of  $I$ 's initial position  $i$  the speculator's actions are shown to reduce efficiency, we can say only that this is the case for such  $i$  measured *relative* to the existing limits on short sales. While for analytical simplicity we do not formally restrict  $I$  from any level of short selling, it turns out that it is never necessary for  $I$  to sell more than two shares in any of the equilibria we derive. Thus, he never needs to sell more than one share short, and there is no effective asymmetry in the two players' ability to short sell.

After the players place their orders, a risk-neutral market maker sees only the net order flow,  $Q$ , and then prices the trades at the risk-neutral expected value given his inference about  $I$ 's signal from observing  $Q$ , as in Kyle (1985). We represent this price as  $p(Q)$ . We assume that the market maker holds sufficient inventory to satisfy any relevant pattern of trades.

Next,  $D$  makes his accept/reject decision based on any information he can learn from the stock price, given that he knows the game being played. The risk-neutral  $I$  would like  $D$  to accept as long as the posterior probability of the good state is at least  $\frac{d-\epsilon}{2d-\epsilon} < \frac{1}{2}$ . However, we assume that  $D$  is risk averse to the extent that he will accept only if his posterior is that the probability of the good state is at least  $\pi$ , where  $\pi \in (\frac{1}{2}, \frac{1}{3} + \frac{2}{3}\lambda]$  captures the severity of the agency problem.<sup>10</sup> The reasons for these specific bounds on  $\pi$  are discussed later where appropriate. Since  $D$  is an individual, while his decision impacts firm value, we assume his overall utility is negligible relative to that of the risk-neutral shareholders of the firm. Thus, we always measure the efficiency of the decision from the point of view of the shareholders.<sup>11</sup>

<sup>10</sup> As noted previously, the same effect could be captured by private benefits or private costs for the decision maker. Also, the qualitative results of the model would be similar in the absence of any agency problem, though the analysis is less tractable. Further details are available upon request.

<sup>11</sup> We do not consider how any surplus arising from the relationship is divided between the firm and the counterparty on whose behalf  $D$  makes the relationship decision. Our measure of efficiency remains valid as long as a valuable relationship for the firm does not create losses for the owners of the counterparty.

After the decision is made, the state of nature and resulting firm value are realized. Finally, all stock positions are closed out—long positions are paid the firm value per share, and those with short positions must pay the firm value per share.

### 3 Equilibrium

For the base model we consider only sequential equilibria with pure trading strategies.<sup>12</sup> We also require that the posterior beliefs of  $D$  and the market maker about the probability of the good state be weakly increasing (monotonic) in net order flow (including those order flows that do not occur in equilibrium).<sup>13</sup> Where multiple equilibria may exist, we focus on the most efficient ones, i.e., those where firm value is maximized because the relationship is correctly accepted with the greatest probability.

Given that an M signal is received with the same probability in the good and bad states, it is uninformative. Thus,  $I$ 's posterior after receiving an M signal is the same as the prior: a  $\frac{1}{2}$  probability of the good state. Since  $\epsilon > 0$ , an acceptance is efficient given this posterior. However, given the lower bound on  $\pi$  specified above, this is insufficient to convince  $D$  to accept the relationship. Note that this therefore justifies the assumed lower bound of  $\pi$  as  $D$ 's risk aversion would be irrelevant if we had  $\pi \leq \frac{1}{2}$ .

<sup>12</sup> Mixed trading strategies are necessary only when we extend the model to an earlier trading round to show that it is rational for the speculator to follow the strategy we derive. See Section 4 for details.

<sup>13</sup> This assumption rules out “perverse” equilibria, such as those in which  $I$  buys more shares after observing an L signal than after observing an H signal, which would mean that prices would actually *decrease* in net order flow over some range. Such equilibria are possible only because of the discrete nature of our modeling assumptions. Other signaling models that use similar monotonicity assumptions to refine the equilibrium set include Bikhchandani (1992), Gul and Sonnenschein (1988), and Swinkels (1999).

The posterior after observing an H signal, using Bayes' rule, is

$$Pr[\Theta = G|\theta = H] = \frac{Pr[\theta = H|\Theta = G]}{Pr[\theta = H|\Theta = G] + Pr[\theta = H|\Theta = B]} = \frac{\lambda}{\lambda + (\frac{1}{2} - \lambda)} = 2\lambda > \frac{1}{2}. \quad (1)$$

Similarly, the posterior after observing an L signal is

$$Pr[\Theta = G|\theta = L] = \frac{Pr[\theta = L|\Theta = G]}{Pr[\theta = L|\Theta = G] + Pr[\theta = L|\Theta = B]} = \frac{\frac{1}{2} - \lambda}{(\frac{1}{2} - \lambda) + \lambda} = 1 - 2\lambda < \frac{1}{2}. \quad (2)$$

We assume

$$V_L \equiv 1 + (1 - 2\lambda)d - 2\lambda(d - \epsilon) < 1, \quad (3)$$

that is, acceptance is inefficient given an L signal. Thus, from  $I$ 's point of view, a fully efficient equilibrium is one in which  $D$  always accepts after an H or M signal, but never after an L.

It is useful to define other values analogously as follows:

$$V_M \equiv 1 + \frac{1}{2}d - \frac{1}{2}(d - \epsilon) = 1 + \frac{1}{2}\epsilon \quad (4)$$

is expected firm value per share if the decision maker accepts when  $\theta = M$ ; and

$$V_H \equiv 1 + 2\lambda d - (1 - 2\lambda)(d - \epsilon) \quad (5)$$

is expected firm value per share if  $D$  accepts when  $\theta = H$ . Finally, note that if an agent's posterior is that there is a  $\frac{1}{3}$  chance the signal is H and a  $\frac{2}{3}$  chance the signal is M, then the posterior probability of the good state is

$$\frac{1}{3}(2\lambda) + \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3} + \frac{2}{3}\lambda. \quad (6)$$

This corresponds to the maximum of the specified range for  $\pi$ , and thus is always sufficient for acceptance. We define

$$V_P \equiv 1 + \left(\frac{1}{3} + \frac{2}{3}\lambda\right)d - \left(\frac{2}{3} - \frac{2}{3}\lambda\right)(d - \epsilon) \quad (7)$$

as the expected firm value per share with an acceptance given that posterior.

We next introduce notation for the posterior beliefs of the market maker and  $D$  for different net order flows. In equilibrium, it does not matter whether  $D$  observes the net order flow or just the price (the one is as good as the other in terms of inferring signal probabilities), so we assume without loss of generality that he can observe the net order flow. As such, the two agents' posterior beliefs are always equivalent. Let  $Q = q_S + q_I + q_N$  denote the realized net order flow given trading quantities of  $q_S$  for the speculator (if she arrives),  $q_I$  for the blockholder, and  $q_N$  for the noise trader. Throughout, for each possible equilibrium we also use the notation  $q_I^H$ ,  $q_I^M$ , and  $q_I^L$  for  $I$ 's *equilibrium* signal-contingent trades. We denote the posterior belief about the probability of state G given  $Q$  as  $\mu(Q)$ .

### 3.1 Efficient equilibria

Now consider the necessary characteristics of a fully efficient equilibrium, in which  $D$  always accepts after an H or M signal and always rejects after an L. The following requirements are immediate (proofs not in the text are in the Appendix).

**Lemma 1** *Any fully efficient pure strategy equilibrium must be such that  $I$  plays the same strategy after M and H signals ( $q_I^M = q_I^H$ ), and plays a sufficiently different strategy after an L signal so that no possible net order flows following that trade could arise from his equilibrium trade after an M or H signal.*

If these conditions are violated, then there must be equilibrium order flows where the efficient decision is not taken. If  $I$  plays different pure strategies after H and M signals ( $q_I^M \neq q_I^H$ ), then some order flows can occur only following an M, and  $D$  concludes upon seeing those order flows that the signal cannot be H and rejects due to his risk aversion (i.e.,  $\mu(Q) < \frac{1}{2} < \pi$  at those order flows). Similarly, if  $I$  plays a strategy after an L signal where the resulting order flow can also follow an M or H, when that order flow occurs either  $D$  sometimes accepts after an L (if the relative probability of an H signal is high enough) or sometimes rejects after an M or H.

We next determine when such fully efficient equilibria exist for both the no speculator and the active speculator cases. In the active speculator case, the speculator's incentive is to trade in the direction of her initial position, i.e., to buy if long and sell if short. This is because the main tension in the model is whether  $D$  will accept after an M signal, and since the relationship is positive-NPV the speculator would like to have it accepted if long (and so buys) and rejected if short (and so sells). Thus, as an additional equilibrium selection criterion (along with monotonicity of beliefs and choosing the most efficient available equilibria), we focus on equilibria where the speculator buys a share if initially long and sells a share if initially short. This allows us to focus on the most interesting cases, where the speculator's scope for disruption is maximized.

For the no speculator case, consider the class of potential equilibria where  $I$  trades a quantity  $q_I^M = q_I^H = q_I^+$  after an M or H signal, and trades  $q_I^L \leq q_I^+ - 3$  after an L signal. The L trade needs to differ by at least three shares so that after a buy from the noise trader it cannot be confused with an M or H signal trade with a sell from the noise trader. The possible equilibrium order flows after an M or H signal are  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , which occur with equal probability from  $I$ 's perspective. After an L signal they are  $Q \in \{q_I^L + 1, q_I^L - 1\}$ , again with equal probability. This class of equilibria represents all possible pure strategy fully efficient equilibria in the no speculator case given our condition that beliefs must be monotonic in order flow (i.e.,  $q_I^L \leq q_I^M \leq q_I^H$  must hold in equilibrium).



The structure of this type of equilibrium, with  $q_I^L = q_I^+ - 3$ , is illustrated in Fig. 1, which shows the prescribed trading quantities for the different signals, the possible resulting net order flows at the ends of the arrows (with probabilities along the arrows determined by the noise trader's buying or selling one share with equal probability), and the resulting equilibrium (and assumed out-of-equilibrium) prices described below. Equilibrium order flows and prices are in bold italics, and out of equilibrium quantities are in normal text.

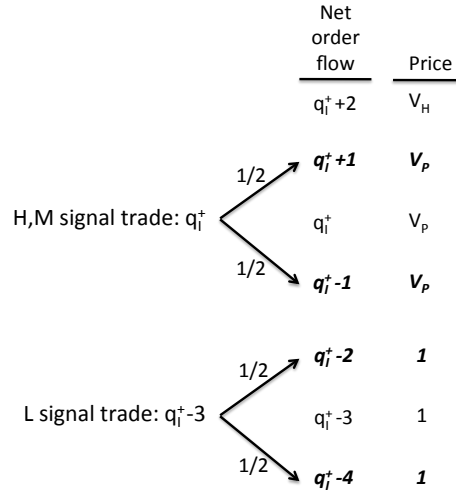


Fig. 1. Proposed equilibrium orders for  $I$ , resulting net order flows, and prices in the no speculator case. Arrows represent the noise trader's random purchase or sale of one share.  $V_H$  equals expected firm value when  $D$  accepts and the signal is H, while  $V_P$  equals expected firm value when  $D$  accepts and the H and M signals are pooled. Equilibrium order flows and prices are in bold italics, while out-of-equilibrium quantities are in normal text.

To understand the prices given in the figure, note that any order flow that can follow an L signal, i.e.,  $Q \in \{q_I^+ - 4, q_I^+ - 2\}$ , results in the belief that the signal was L, which leads to rejection and a value of one. Using Bayes' Rule, any order flow that can follow an M or H, i.e.,  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , results in the belief that there is a  $\frac{1}{3}$  probability that the signal was H, and a  $\frac{2}{3}$  probability it was M.<sup>14</sup> The posterior is thus  $\frac{1}{3} + \frac{2}{3}\lambda$ , which leads  $D$  to accept since  $\pi \leq \frac{1}{3} + \frac{2}{3}\lambda$

<sup>14</sup>To see this, note that  $I$  receives an M signal with unconditional probability  $\frac{1}{2}$ , and an H signal with unconditional probability  $\frac{1}{4}$  (the state is good with probability  $\frac{1}{2}$  leading to an H signal with probability  $\lambda$ , and the state is bad with probability  $\frac{1}{2}$  leading to an H signal with probability  $\frac{1}{2} - \lambda$ , so the unconditional probability of an H signal equals  $\frac{1}{2}\lambda + \frac{1}{2}(\frac{1}{2} - \lambda) = \frac{1}{4}$ ). Thus, when  $D$  believes that  $I$  is pooling after M and H signals and he observes a corresponding order flow, he concludes that the signal was H with probability  $\frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$ .

(note that this also justifies the upper end of the allowed range for  $\pi$ —if it were higher, even full pooling between M and H signals would be insufficient for acceptance). Since the market maker believes that  $D$  will accept, and has the same posterior belief about the probability of the good state, he sets the price at  $p(Q) = V_P$  for such order flows. Two of the off-equilibrium prices shown (at order flows of  $q_I^+$  and  $q_I^+ - 3$ ) are pinned down by our assumption of monotonic beliefs, while the price of  $V_H$  at  $q_I^+ + 2$ , reflecting the belief that the signal is H for all order flows at or above  $q_I^+ + 2$ , turns out to be the most effective assumption for preventing upward deviations by  $I$  following an H signal. Similarly, setting a belief that the signal is L at all lower order flows, resulting in a price of one, is the most effective assumption for preventing downward deviations by  $I$  after an M signal. Since upward deviations after an H and downward deviations after an M are the hardest deviations to prevent, this set of off-equilibrium prices ensures existence over the largest possible parameter space.

Now consider the blockholder's expected trading profits. After an M signal  $I$  knows that the expected per share value is actually  $V_M$  if  $D$  accepts, so he expects to take a loss equal to  $q_I^+(V_M - V_P)$ . On the other hand, after an H signal he expects a gain equal to  $q_I^+(V_H - V_P)$ .

These trading gains and losses lead to a friction between pooling and separating that makes it difficult to sustain fully efficient equilibria. First, following an M signal  $I$  may not be willing to suffer the trading losses at the pooled price, so may deviate downward to a smaller trade. This will cause a loss with respect to the value of his initial position,  $i$ , since an efficient acceptance becomes less likely, but will save (at least some of) the potential trading loss. This type of deviation will be more likely the smaller is his initial position  $i$ , i.e., the less  $I$  cares about the ultimate firm value. On the other hand,  $I$  may want to deviate upward from the pooled quantity in order to increase his trading gains following an H signal. The size of his initial position is less of an issue here since  $D$  always accepts at higher order flows (so  $I$  need not worry about an inefficient decision if he deviates upward).

The latter effect results in a minimum quantity that the blockholder will accept as an equilibrium pooled trade after an H signal. To see this, consider a deviation trade of  $q_I^+ + 2$ , which turns out to be the most profitable deviation. If the noise trader buys, the resulting order flow is  $q_I^+ + 3$ , which reveals the H signal and results in zero trading profit. If the noise trader sells, the order flow is  $q_I^+ + 1$ , the deviation is hidden, and the blockholder gets trading profits of  $(q_I^+ + 2)(V_H - V_P)$ . Since the relationship is accepted either way, the blockholder will deviate if  $\frac{1}{2}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_P) \implies q_I^+ < 2$ .

This minimum trading quantity creates a problem for the blockholder when his signal is M, because with  $q_I^+ = +2$ , he must take equilibrium trading losses of  $2(V_M - V_P)$ . His best deviation in this case would be to not trade, which saves these losses but causes rejection  $\frac{1}{2}$  of the time when the noise trader sells. That would reduce the value of his stake from  $iV_M$  to  $i(\frac{1}{2}V_M + \frac{1}{2})$ , so this deviation is profitable if  $iV_M + 2(V_M - V_P) < i(\frac{1}{2}V_M + \frac{1}{2}) \implies i < \frac{4(V_P - V_M)}{V_M - 1}$ . Thus, when this condition holds, there is inefficiency due to the loss of some positive-NPV relationships even in the absence of the speculator.

Next consider the active speculator case. To understand the role that the speculator plays, note that she effectively adds strategic “noise” to the system, as only she knows her starting position and thus how she will trade. This has several effects. First of all, it means that  $I$  will have to spread his signal-contingent trades wider in order to fully separate his L signal trade from his M and H signal trade. In other words,  $I$  will either have to sell more after an L, buy more after an M or H, or both. Second, the additional uncertainty impacts both of the deviation incentives described above in a way that makes fully efficient equilibria harder to support. In particular, it makes both downward deviations after an M signal and upward deviations after an H signal *more* profitable because the deviations become harder to detect.

To see this, consider the class of equilibria where  $I$  trades  $q_I = q_I^+$  after an M or H signal (as above), but now trades  $q_I = q_I^L \leq q_I^+ - 5$  after an L signal to ensure full separation. The

difference required for separation increases to five shares because the speculator's trades expand the range of "noise" from two to four shares (the net "noise" trade is now +2, 0, or -2 shares with respective probabilities  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$ ). The proposed equilibrium is illustrated in Fig. 2, assuming  $q_I^L = q_I^+ - 5$ . Again, equilibrium quantities are in bold italics, and out-of-equilibrium quantities are in normal text. Out-of-equilibrium beliefs (and prices) are either pinned down by our monotonicity assumption, or, as above, set to the extreme highs for higher order flows or extreme lows for lower order flows. As with the no speculator case, this class of equilibria is the only possible class of pure strategy fully efficient equilibria in the active speculator case.

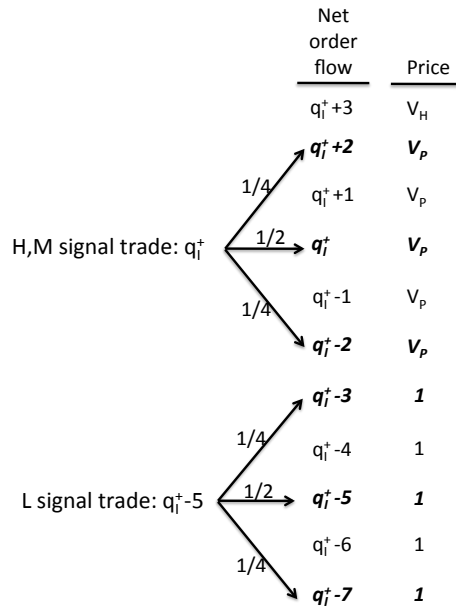


Fig. 2. Proposed equilibrium orders for  $I$ , resulting net order flows, and prices in the active speculator case. Arrows represent the net effect of the noise trader's and speculator's random purchase or sale of one share each.  $V_H$  equals expected firm value when  $D$  accepts and the signal is H, while  $V_P$  equals expected firm value when  $D$  accepts and the H and M signals are pooled. Equilibrium order flows and prices are in bold italics, while out-of-equilibrium quantities are in normal text.

To see why it is harder to sustain such equilibria, note that if the blockholder deviates up by two shares after an H signal, the deviation will be detected only with probability  $\frac{1}{4}$  (when the order flow is  $q_I^+ + 4$ ), leaving trading profits with probability  $\frac{3}{4}$ . This makes the deviation significantly more profitable, implying a disproportionately higher required trade, as seen in the condition for profitable deviation:  $\frac{3}{4}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_P) \implies q_I^+ < 6$ . But at a trade size of

+6, the blockholder must take larger losses after an M signal, so deviations will again be more profitable. For instance, if he deviates down to a trade of +4, his deviation will be detected and the relationship rejected with only  $\frac{1}{4}$  probability, but he will reduce his trading losses. This deviation is profitable if  $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}4(V_M - V_P) > iV_M + 6(V_M - V_P) \implies i < \frac{12(V_P - V_M)}{V_M - 1}$ . Note that the block size necessary with the speculator is three times larger than that required with no speculator, even though the speculator trades only one share.

Checking all other possible deviations (see the Appendix), yields the following result.

**Theorem 1** *In the no speculator case, a fully efficient equilibrium in which  $q_I^H = q_I^M \geq 2$ ,  $q_I^L \leq q_I^M - 3$ , and  $D$  accepts the relationship iff  $Q \geq q_I^M - 1$ , exists if  $i > i^{*N} = \frac{4(V_P - V_M)}{V_M - 1}$ . Otherwise, no fully efficient equilibria exist.*

*In the active speculator case, a fully efficient equilibrium in which  $q_I^H = q_I^M \geq 6$ ,  $q_I^L \leq q_I^M - 5$ ,  $S$  buys one share if long and sells one share if short, and  $D$  accepts the relationship iff  $Q \geq q_I^M - 2$ , exists if  $i > i^{*S} = \frac{12(V_P - V_M)}{V_M - 1}$ . Otherwise, no fully efficient equilibria exist.*

*Finally, we clearly have  $i^{*S} > i^{*N}$ .*

This result implies that there is a large range of stake sizes for which no fully efficient equilibria exist with an active speculator, but do exist without. Thus, the actions of the speculator are likely to reduce efficiency in this region.

However, in the range where full efficiency exists, an active speculator has no impact on the decision maker. Thus, it is straightforward to show that, while an active trading strategy in a fully efficient equilibrium can be incentive-compatible for the speculator, it will not generate any profit. It will be incentive-compatible because, from the speculator's perspective, all of her trades are at zero profit or zero loss. The only other possible source of profit is an increase in the value of her initial position, but in a fully efficient equilibrium her presence does not affect overall firm value, so no profit occurs. To determine whether the speculator will ever profit from

actively trading, we need to determine what type of equilibria may exist over ranges *without* fully efficient equilibria, and whether any such equilibria support profitable speculation.

### 3.2 *Partial pooling equilibria*

We continue the strategy of first determining the most efficient possible equilibrium, and then checking for its existence. One possible class of equilibria is one in which the H signal is fully separated from the M and L signals. In fact, there are always equilibria where  $I$  trades a large positive amount after an H signal, and trades amounts after M or L signals that fully separate them from the trade following an H.<sup>15</sup> However, there are some intermediate equilibria that are both more efficient and allow for potential profits for the speculator. In particular, we characterize the existence of pure strategy “partial pooling” equilibria in which  $D$  always accepts after an H, never accepts after an L, and sometimes accepts after an M.

For now assume again that  $I$  is always willing to separate after an L signal to ensure a rejection (verified in the Appendix). In order to have an equilibrium where  $D$  sometimes accepts after an M,  $I$ 's trades after M and H signals must be separated by a multiple of two, with the smaller trade following the M signal. If that were not the case, then the resulting order flows could never coincide because the strategy would always result in odd net order flows after one signal, and even net order flows after the other. For the no speculator case, overlap is possible only with a 2-share difference, so the only possible partial pooling equilibrium is one in which  $I$  trades two fewer shares after an M signal than after an H. In this case, we have the following result.

<sup>15</sup>This results in acceptance only after an H signal, so the equilibrium trades after M and L signals do not matter either for relationship outcomes or for trading profits as long as the resulting net order flows cannot overlap with those following an H. Some of these equilibria thus have the M and L signals separated from each other, and some do not. Since there is no trading gain or loss after M or L signals in equilibrium (the price is always correctly  $p(Q) = 1$ ), existence of these equilibria depends on the blockholder's stake being small enough relative to the trade following an H signal so that  $I$  will not try to deviate up after an M signal.

**Proposition 2** *Assume the speculator is not active. Then for all  $i \leq i^{*N}$ , a partial pooling equilibrium exists in which  $q_I^M \geq 0$ ,  $q_I^H = q_I^M + 2$ ,  $q_I^L < q_I^M - 1$ , and  $D$  accepts the relationship iff  $Q \geq q_I^M + 1$ .*

Now consider the active speculator case. We continue to focus on equilibria in which the speculator buys if initially long and sells if initially short. The conditions under which this is optimal for the derived equilibria are given in the proof of Theorem 3 in the Appendix. For the remainder of the paper, we focus on the part of the parameter space where the relevant incentive-compatibility constraints can be satisfied by an initial position for the speculator of magnitude  $s = 1$ , as this is the interesting part of the parameter space when noise trade has magnitude 1.<sup>16</sup>

With an active speculator, there are two possible partial pooling equilibria, one with a 2-share difference and one with a 4-share difference between the trades after M and H signals. It turns out that whether a 4-share difference or a 2-share difference is more efficient depends on the value of  $\pi$ , i.e., the extent of  $D$ 's risk aversion. This is because the outcomes in an equilibrium with a 2-share difference depend on which of two ranges  $\pi$  falls into. Fig. 3 provides an illustration of  $I$ 's trades in such an equilibrium after M and H signals, with resulting possible order flows and posterior beliefs. Note that if  $\pi \in (\frac{2}{5} + \frac{2}{5}\lambda, \frac{1}{3} + \frac{2}{3}\lambda]$ , the relationship will be rejected at the node where the order flow is  $q_I^M$  because the posterior is insufficient. Thus, the relationship is accepted with probability  $\frac{3}{4}$  after an H signal, and with probability  $\frac{1}{4}$  after an M (i.e., only if both the speculator and noise trader buy so that  $Q = q_I^M + 2$ ). However, if  $\pi \in (\frac{1}{2}, \frac{2}{5} + \frac{2}{5}\lambda]$ ,  $D$  will accept at all order flows that can follow an H signal (i.e., all  $Q \geq q_I^M$ ), so this equilibrium has acceptance with probability 1 after an H signal and with probability  $\frac{3}{4}$  after an M.

The efficiency of an equilibrium with a 4-share difference, on the other hand, is not affected by the exact level of risk aversion. Since net order flows following an H and M signal overlap only at

<sup>16</sup> Technically, this parameter restriction corresponds to the assumption that  $\frac{2(V_P - V_M)}{V_M - 1} < 1$  when  $\pi \in (\frac{2}{5} + \frac{2}{5}\lambda, \frac{1}{3} + \frac{2}{3}\lambda]$ , and  $\frac{27(V_P - V_M)}{V_M - 1} < 1$  when  $\pi \in (\frac{1}{2}, \frac{2}{5} + \frac{2}{5}\lambda]$ .

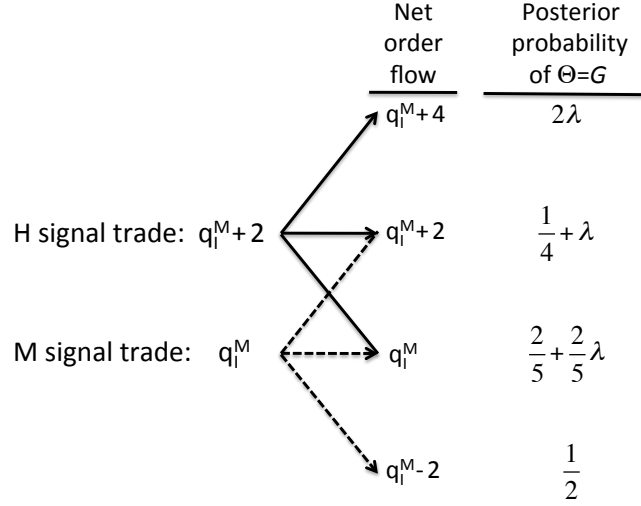


Fig. 3. Equilibrium orders, resulting net order flows, and posterior beliefs for a partial pooling equilibrium with a 2-share difference. Arrows represent the net effect of the noise trader’s and speculator’s random purchase or sale of one share each. The parameter  $\lambda \in (\frac{1}{4}, \frac{1}{2})$  measures the informativeness of  $I$ ’s signal.

one node (after an H signal when both  $S$  and the noise trader sell, and after an M signal when both  $S$  and the noise trader buy), and the posterior at that order flow is  $\frac{1}{3} + \frac{2}{3}\lambda$ , the relationship is accepted at that node for any allowed  $\pi$ . Thus, in such an equilibrium the relationship is always accepted after an H signal and is accepted with probability  $\frac{1}{4}$  after an M. This is more efficient than the 2-share equilibrium when risk aversion is in the higher range, but less efficient when risk aversion is in the lower range. The following result establishes the existence of the corresponding “most efficient” partial pooling equilibrium for each range of  $\pi$ .

**Theorem 3** Assume  $\pi \in (\frac{2}{5} + \frac{2}{5}\lambda, \frac{1}{3} + \frac{2}{3}\lambda]$ . Then for all  $i \in [1, i^{*S}]$  a partial pooling equilibrium exists in which  $q_I^M \geq 1$ ,  $q_I^H = q_I^M + 4$ ,  $q_I^L < q_I^M$ ,  $S$  buys one share if long and sells one share if short, and  $D$  accepts the relationship iff  $Q \geq q_I^M + 2$ . The speculator’s strategy entails an expected trading loss of  $\frac{1}{4}(V_P - V_M)$ , but is profitable in expectation because it increases the expected value of her initial position by  $\frac{1}{8}s(V_M - 1) > \frac{1}{4}(V_P - V_M)$ .

Assume  $\pi \in (\frac{1}{2}, \frac{2}{5} + \frac{2}{5}\lambda]$ . Then for all  $i \in [1, i^{*S}]$  a partial pooling equilibrium exists in which  $q_I^M \geq 1$ ,  $q_I^H = q_I^M + 2$ ,  $q_I^L < q_I^M - 2$ ,  $S$  buys one share if long and sells one share if short, and  $D$



accepts the relationship iff  $Q \geq q_I^M$ . The speculator's strategy entails an expected trading loss of  $\frac{27}{80}(V_P - V_M)$ , but is profitable in expectation because it increases the expected value of her initial position by  $\frac{1}{8}s(V_M - 1) > \frac{27}{80}(V_P - V_M)$ .

It turns out that these partial pooling equilibria give the speculator an opportunity to profit from manipulation. Since  $S$  is uninformed about the future state, she cannot generate trading profits. Thus, in order to have a profitable strategy she must influence the firm's real value so that she realizes a gain on her initial position. In the above equilibria, the speculator knows that if she sells and the signal is M,  $D$  is more likely to reject. However, if she buys,  $D$  is more likely to accept. This wedge gives her the incentive to trade in the direction of her original position since she can cause an inefficient rejection (if she is short and sells) or make an efficient acceptance more likely (if she is long and buys), leading to an increase in the value of her initial position in either case.

However, the trade itself will take place at a loss. Consider what can happen when the speculator sells in a partial pooling equilibrium with a 4-share difference. On the one hand, the signal may be M or L, so  $D$  will reject and the price will be  $p(Q) = 1$ , the correct price. On the other hand, the signal may be H. If the noise trader buys, this offsets  $S$ 's sell trade and the price is, correctly,  $V_H$ . If the noise trader sells, this reinforces  $S$ 's trade and the price is  $V_P$  which is *too low* from  $S$ 's perspective since she can infer from the net order flow that the signal must be H. Thus,  $S$  sells at too low of a price and faces an expected trading loss. A similar argument shows that if  $S$  buys she will either do so at zero trading profit, or at a trading loss due to buying at too high of a price,  $V_P$ , since  $S$  can infer that  $I$ 's signal is M. Similar logic holds for her trades in an equilibrium with a 2-share difference.

This implies that in order for active speculation to be incentive-compatible (and profitable), the speculator must have a sufficiently large initial position so that the gain on that position will overcome the potential trading losses. As noted above, we have focused on the part of the

parameter space where a position of magnitude  $s = 1$  is sufficient for this purpose. Thus, for this part of the parameter space the result confirms that active speculation can be profitable even for an otherwise uninformed speculator who has accumulated a “secret” long or short position in the stock, or an effective long or short position through holdings in correlated instruments such as the securities of a competitor, supplier, customer, or counterparty.

### 3.3 Efficiency implications

The results thus far imply that a speculator’s presence can reduce efficiency by causing an inefficient rejection by  $D$ . An informed long-term blockholder can prevent this loss, but at an endogenous cost that cannot be justified when his own initial position is not sufficiently large. These results are summarized in Fig. 4 below, which plots the corresponding “most efficient” equilibrium as a function of  $I$ ’s initial position  $i$ .

The rightward arrow in each panel of the figure represents increasing values of  $I$ ’s initial position,  $i$ . The labeled values correspond to the thresholds from Theorem 1. For values of  $i$  above  $i^{*S}$ , a fully efficient equilibrium exists with or without the speculator. For values of  $i$  from  $i^{*N}$  to  $i^{*S}$ , a fully efficient equilibrium exists without the speculator, while the most efficient equilibrium in that range with an active speculator is a partial pooling equilibrium in which  $D$  accepts either  $\frac{1}{4}$  or  $\frac{3}{4}$  of the time after an M signal. Thus, positive-NPV relationships are rejected some of the time. Also, in this range an active speculator can make profits if her initial position is at least one share, since her trading activity can induce inefficient decisions.

Furthermore, the most efficient partial pooling equilibrium with an active speculator given  $\pi > \frac{2}{5} + \frac{2}{5}\lambda$  (which has acceptance after an M signal  $\frac{1}{4}$  of the time) is less efficient than the most efficient one that exists given  $\pi < \frac{2}{5} + \frac{2}{5}\lambda$  (which has acceptance after an M signal  $\frac{3}{4}$  of the time). Thus, greater risk aversion on the part of the decision maker leads to greater efficiency loss.

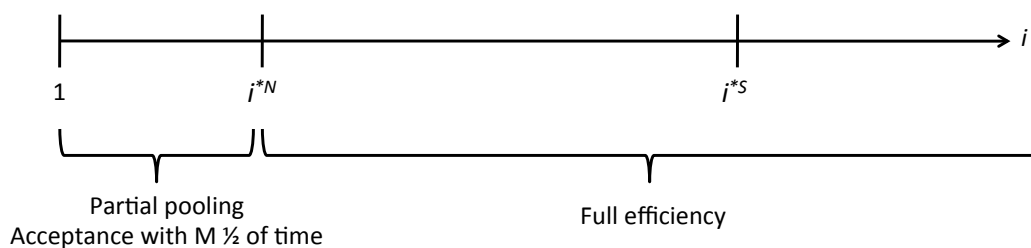
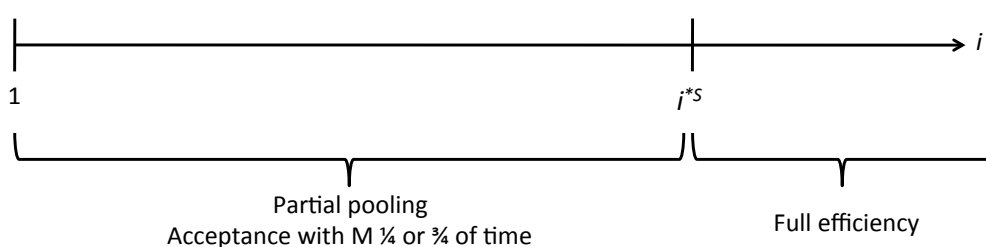
No speculator caseActive speculator case

Fig. 4. Equilibrium map showing the most efficient equilibrium as a function of  $I$ 's stake size.

When  $i$  is between one and  $i^{*N}$ , the most efficient equilibrium both with and without the speculator is a partial pooling equilibrium. As expected, for most of the parameter space the speculator's actions result in a reduction in the maximum attainable firm value.<sup>17</sup> Recall that these results are best interpreted in relative terms since, for the model so far, we have restricted the speculator to trading at most one share. In particular, the results show that uninformed speculators can (profitably) affect firm value if the blockholder's stake is not large enough *relative* to the existing level of constraints on short selling.

<sup>17</sup> The only exception is that for lower levels of risk aversion such that  $\pi < \frac{2}{5} + \frac{2}{5}\lambda$ , the partial pooling equilibrium can actually be more efficient with an active speculator. However, this is due mainly to the fact that we assume discrete trading quantities. Because of this assumption, there is only one possible type of partial pooling equilibrium in the absence of the speculator, and it happens to be more efficient than the 4-share equilibria with a speculator and less efficient than the 2-share equilibria with a speculator. Thus, this result arises from our specific modeling choices, not the general economics of the problem.

Clearly, the lower the thresholds illustrated in Fig. 4, the more likely it is that blockholders already hold sufficient stakes and successful bear raids will be prevented. An important question then is how these thresholds vary with the importance of decisions “at risk” from a bear raid. In this respect, consider the effect of an increase in  $\epsilon$ . This increases the expected value of the relationship given both H and M signals, but in particular following M signals (the derivative of  $V_M$  with respect to  $\epsilon$  is  $\frac{1}{2}$ , while the derivative of  $V_H$  with respect to  $\epsilon$  is  $(1 - 2\lambda) < \frac{1}{2}$ ). Thus, an increase in  $\epsilon$  can be interpreted as an increase in the importance of the decision following an M signal, and as a measure of how important it is to prevent a successful bear raid. We have the following comparative static.

**Proposition 4** *The thresholds  $i^{*N}$  and  $i^{*S}$  are decreasing in  $\epsilon$ .*

An increase in  $\epsilon$  increases the blockholder’s incentive to prevent bear raids because it increases the benefit to firm value from inducing an acceptance after an M signal, making him more willing to incur the necessary expected trading losses. However, this intuitive effect is not the only relevant one. It also turns out the expected trading losses he must suffer *decrease* because  $V_M$  is increasing in  $\epsilon$  faster than is  $V_H$ , and thus the spread between the pooled price  $V_P$  and the expected value  $V_M$  decreases. When combined, these two (reinforcing) effects imply that deviations from the efficient equilibrium become less attractive, reducing the size of the block needed. Thus, blockholders are better placed to prevent bear raids precisely when they are likely to be most damaging.

#### 4 The speculator’s initial position and overall profits

In this section we introduce an additional trading round (hereafter the “first round”) that takes place prior to the base model trading round (hereafter the “final round”). This allows us to endogenize the stake the speculator chooses before entering the final round. As noted previously,

$S$ 's trades in the final round are executed at an expected loss, so for her trades in that round to be incentive-compatible it is necessary that she be able to arrive at that stage with a sufficiently large (and secret) position. Here we derive a mixed strategy equilibrium for the first round that allows her to arrive at the final round with such a position, and show that the overall strategy over the two rounds can be profitable.

Like in the final round, we assume that in the first round the noise trader buys one share with probability  $\frac{1}{2}$  and sells one share with probability  $\frac{1}{2}$ . At the time of this round we assume that  $I$  has not yet received his signal  $\Theta$ . Since we consider  $I$  to be a long-term investor whose optimal stake,  $i$ , is determined by factors outside our model (e.g., diversification concerns), we assume he does not alter his stake size in this round.<sup>18</sup> The speculator arrives at the first round with no position in the stock and can buy or sell one share, or choose not to trade. The market maker observes net order flow and sets the price at the risk-neutral expected value (with full knowledge of the potential implications of that order flow for the final round). For parsimony, we only consider the case where  $\pi \in (\frac{2}{5} + \frac{2}{5}\lambda, \frac{1}{3} + \frac{2}{3}\lambda]$ , so that the most efficient partial pooling equilibrium has a 4-share difference between the trades following M and H signals.

In the first round the speculator must play a mixed strategy to make trading profits, since otherwise she will always arrive in the final round with a known position. In particular, we show that it is an equilibrium strategy for  $S$  to buy one share with probability  $\frac{1}{2}$  and sell one share with probability  $\frac{1}{2}$ . This obviously corresponds to the base model's assumptions by equating  $s = 1$ ,  $s$  being the magnitude of the speculator's position upon entering the final round.

If the speculator mixes in this way, her position will be hidden (secret) only  $\frac{1}{2}$  of the time—when the noise trader trades in the opposite direction. When their trades are reinforcing, the speculator's trade will be revealed. Thus, in order to prove that  $S$ 's mixed strategy is part of

<sup>18</sup> We have also analyzed the case where the blockholder is allowed to optimally adjust his stake in the first round, but faces diversification costs for holding a suboptimally large stake between rounds. Our main results continue to hold, and the analysis is available upon request.

an overall equilibrium, we must first identify what happens in the final round when  $S$ 's position has been revealed.

In such cases, we show that a pure strategy equilibrium exists in which  $S$  trades a single share in the same direction as her first round trade, while everyone else essentially behaves as in the no speculator case. In other words, if the speculator arrives with a known position and is expected to play a pure strategy in the final round, the market maker and decision maker are able to net out her effect on the order flow, and the equilibrium outcomes are the same as with no speculator. Since  $S$ 's trades do not affect any outcomes, they are priced correctly (from her perspective) and entail no trading loss.

**Lemma 2** *Assume  $S$ 's position on entering the final round is common knowledge. Then:*

*If  $i \geq i^{*N}$ , there exists an equilibrium of the final round in which  $S$  buys one share if long and sells one share if short,  $q_I^H = q_I^M \geq 2$ ,  $q_I^L \leq q_I^M - 3$ , and  $D$  accepts the relationship iff  $Q \geq q_I^M - 1 + q_S$ .*

*If  $i < i^{*N}$ , there exists an equilibrium of the final round in which  $S$  buys one share if long and sells one share if short,  $q_I^M \geq 0$ ,  $q_I^H = q_I^M + 2$ ,  $q_I^L < q_I^M - 1$ , and  $D$  accepts the relationship iff  $Q \geq q_I^M + 1 + q_S$ .*

*In both cases  $S$ 's expected trading profit is zero.*

From here on we assume these equilibria form the subgame following outcomes where  $S$ 's first round trade is revealed by a reinforcing noise trade, whereas the most efficient available full efficiency and partial pooling equilibria derived in Section 3 for the active speculator case form the subgame following outcomes where  $S$ 's first round trade is hidden. This gives us the following result.

**Theorem 5** *A first round equilibrium exists in which  $S$  buys one share with probability  $\frac{1}{2}$  and sells one share with probability  $\frac{1}{2}$ . If  $i < i^{*S}$  and the noise trader trades in the opposite direction from  $S$ , the stock price is  $\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H$  and  $S$  makes an overall expected profit of  $\frac{1}{8}(V_M - 1) - \frac{1}{4}(V_P - V_M) > 0$ . If  $i \geq i^{*S}$  or the noise trader trades in the same direction as  $S$ ,  $S$ 's overall expected profit is zero.*

This result shows that the speculator is able to profit in expectation as long as her first round trade is hidden and  $i$  is such that the blockholder does not hold a large enough stake to ensure full efficiency, i.e., if a partial pooling equilibrium prevails in the final round. The first round trade is profitable because the trade is priced at an average of the expected value of the firm with a long versus short speculator. There is a gap between these values since, in the partial pooling equilibrium, the speculator will either induce an increase in firm value on average (if long) or induce a decrease (if short). The speculator plays off this gap, capturing first round expected trading profits that exceed the final round expected trading losses. On the other hand, if the blockholder's stake is above the efficiency threshold or  $S$ 's trade is revealed, then the speculator's first round trades are still incentive-compatible, but her strategy entails an expected profit of zero in both rounds because her trades have no effect on the decision maker.

## 5 Multiple blockholders

Many firms have multiple, smaller blockholders rather than one large blockholder. If the blockholders can coordinate and act as a unit (i.e., commit among themselves to a joint trading strategy), then the base model applies with the stake size  $i$  set equal to the sum of the blockholders' stakes. However, if they cannot commit to a unified trading strategy, then having a disaggregated block could make it more difficult to achieve efficiency. In this section we consider an extension of the model with two symmetric blockholders, and ask whether the stake size required for efficiency differs from that for a single blockholder.

For simplicity, we study only the final round and assume that each blockholder  $j \in \{1, 2\}$  enters with a stake size of  $i_j$ , where  $i_1 = i_2$ . We assume the blockholders receive the same signal, but cannot coordinate their trading decisions. One might expect that with multiple blockholders trading against the speculator, each blockholder's required trade would be smaller, and the required cumulative block size might be the same or even smaller than in Theorem 1. Past work on blockholders shows results similar to this when governance is exercised through trading, i.e., in "exit" models such as Edmans and Manso (2011), where, in the absence of coordination, trading by multiple blockholders results in more informative prices. Theories of "voice" on the other hand find that having multiple blockholders reduces the effectiveness of blockholder governance because of increased free-riding. Surprisingly, we find a result more in line with the "voice" theories despite the fact that blockholders govern here through trading, as in the exit models. In fact, we have the following result.

**Proposition 6** *A fully efficient equilibrium, in which blockholder  $j$ 's signal-contingent trades are  $q_{I_j}^H = q_{I_j}^M \geq 6$  and  $q_{I_j}^L$  such that  $q_{I_1}^L + q_{I_2}^L \leq q_{I_1}^M + q_{I_2}^M - 5$ , the speculator buys one share if long and sells one share if short, and  $D$  accepts the relationship iff  $Q \geq q_{I_1}^M + q_{I_2}^M - 2$ , exists if  $i_1 = i_2 \geq i^{*S} = \frac{12(V_P - V_M)}{V_M - 1}$ . If  $i_1 = i_2 < i^{*S} = \frac{12(V_P - V_M)}{V_M - 1}$ , no fully efficient pure strategy equilibria exist.*

Essentially, each informed blockholder faces the same trading incentives as the single blockholder in the base model. Given the expected trades of the other blockholder, each one faces the same expected trading profit/loss per share in equilibrium, and each possible deviation has the same impact on that blockholder's profits as for the single blockholder in the base model. Intuitively, since the blockholders are uncoordinated, each wants to trade just as aggressively to achieve trading profits with an H signal as they would if they were a single blockholder, but this imposes a high cost because of the trading losses that have to be incurred with an M signal.<sup>19</sup> Thus, to

<sup>19</sup> The necessity here of incurring trading losses to govern is similar in spirit to the costs of intervention in voice theories, and partially explains why our result is more in line with these theories.



achieve efficiency, each informed blockholder must hold a stake as large as that required for a single blockholder. In the absence of coordination, efficiency is therefore harder to achieve the more dispersed the block ownership. This result should prove useful for empirical tests of the model in addition to providing guidance for optimal ownership structures.

## 6 Empirical implications

Our model provides a number of new empirical implications. First, it identifies the role of a large blockholder in countering manipulative short selling. Second, it shows that both the trading quantity required *and* the size of the necessary block are disproportionately large relative to the extent of short selling. Furthermore, for a given block size, manipulation is more likely to succeed in destroying value the worse is the agency problem between shareholders and decision makers (e.g., when decision makers are more risk averse with respect to the firm's dealings), when the decisions at risk are less important, and when there are multiple informed blockholders who are unable to coordinate and do not individually hold a large enough stake.

We can derive some additional comparative statics from the thresholds in Proposition 1:

**Proposition 7** *The thresholds  $i^{*N}$  and  $i^{*S}$  are increasing in  $\lambda$  and  $d$ . However, if  $\epsilon = \kappa d$  for some proportion  $\kappa < 1$ , then the thresholds are independent of  $d$ .*

The result with respect to  $\lambda$  implies that the speculator will be more likely to find manipulation profitable if informed shareholders' information is relatively precise (when their signal is, in fact, informative). Intuitively, an increase in  $\lambda$  increases the wedge between the perceived value of an acceptance with an H versus M signal—driving  $V_H$  up while leaving  $V_M$  unchanged. This raises the price  $V_P$  without increasing the incentive for  $I$  to ensure an acceptance after an M signal, i.e., the increase in information precision makes trading profits more important relative to protecting the value of the stake. As a result, it is harder to get  $I$  to pool after an M, i.e.,

pooling requires a larger initial position  $i$ . This has direct empirical implications about which situations are more amenable to manipulation by a speculator. It also implies that an increase in information precision can actually reduce efficiency. If the current block size  $i$  is just above the threshold, then an increase in precision can make it inadequate, causing a discrete shift to a less efficient partial pooling equilibrium.

The result with respect to  $d$  is similar. Increasing  $d$  without changing  $\epsilon$  makes an acceptance more profitable for the firm under an H signal, which drives up the pooled price and makes downward deviation more attractive for  $I$  after an M. Thus, decisions that ex ante have greater profit potential are more likely to encourage speculators to manipulate prices. On the other hand, if  $\epsilon$  and  $d$  are held in strict proportion, a change in decision “scale” (an increase in  $d$  and  $\epsilon$  in lockstep) has no effect on the thresholds. This is because the increased impact of the decision affects  $I$ 's incentive to ensure an acceptance after an M signal and the trading losses required to do so by the same proportion. Overall, these results imply that profitability matters more than scale in terms of predicting when manipulation is likely to destroy value.

Going outside the strict confines of the model, we can provide additional predictions with respect to which types of firms and situations are likely to become more vulnerable to attempted manipulation. First, our model implicitly assumes that opportunities depreciate relatively quickly, i.e., if a relationship is not accepted, the decision cannot be changed later. The problem could clearly be ameliorated if this were not the case and the value loss were less permanent. Second, an unexpected relaxation of restrictions on short selling could create an imbalance between the longs and the shorts, potentially allowing for successful bear raids.

## 7 Conclusion

We argue that the existing debate over the costs and benefits of short selling activity has overlooked an important participant in the market, the long-term blockholder. If there is concern that short sellers can cause an inferior allocation of resources by manipulating prices down, a blockholder has a powerful natural incentive to battle the shorts in an attempt to prevent such undesirable outcomes. However, he also has a potentially conflicting incentive to use his information to generate trading profits. In this setting a speculator can exploit this conflict, forcing the blockholder to buy a disproportionately large amount compared to the expected amount of short selling. Since some of this buying may need to be done at unfavorable prices, it can lead to significant trading losses. If the blockholder's existing stake is insufficient to justify incurring these losses, then short sellers may succeed in inducing suboptimal decisions and destroying value. However, we also show that smaller stakes are sufficient for preventing bear raids when the decisions at risk have larger value impact, which also turn out to be the cases where the costs of preventing bear raids are lower. Thus, markets have some ability to self-correct against the more egregious abuses of short selling.

Our analysis implies that if outside intervention is to be considered, it should be targeted to firms or circumstances where blockholdings are likely to be inadequate. This could arise because of diversification motives; uncertainty about the extent of potential short selling; unexpected or sudden relaxing of market-imposed constraints on short selling through, for instance, non-exchange-traded credit default swap contracts; or simply because the timing of important decisions is stochastic requiring blockholders to hold undiversified positions for indefinite periods. Furthermore, if the possibility of value destruction is considered significant, potential remedies may lie not only in restricting short sellers, but also in enhancing the incentives for blockholders to increase their positions. The paper also points out that existing restrictions on short sales, which make it harder for speculators to amass positions, play an important role.

## Appendix

**Proof of Lemma 1.** Given any pure strategy for  $S$ , if  $q_I^H \neq q_I^M$ , then there will be some order flows  $Q$  after a trade of  $q_I^M$  such that only trades of  $q_I^M$  or  $q_I^L$  could result in those order flows in equilibrium. Thus,  $\mu(Q) \leq \frac{1}{2}$  must hold at those order flows, and  $D$  will not accept since  $\pi \geq \frac{1}{2}$ . With respect to  $I$ 's strategy after an L, if  $q_I^H = q_I^M$  while  $q_I^L$  is such that the resulting equilibrium order flows could not follow a trade of  $q_I^M$ , then all possible equilibrium order flows  $Q$  that *can* result after a trade of  $q_I^M = q_I^H$  will lead to beliefs  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , which is sufficient for acceptance since  $\pi \in (\frac{1}{2}, \frac{1}{3} + \frac{2}{3}\lambda]$ . If instead  $q_I^L$  were such that any of the possible resulting order flows could also result from a trade of  $q_I^M = q_I^H$ , then either the relationship is inefficiently accepted after an L or inefficiently rejected after an M or H. QED

**Proof of Theorem 1.** First consider the no speculator case with the generic class of efficient pure strategy equilibria introduced in the text. We formally specify out-of-equilibrium beliefs as follows. For all  $Q \leq q_I^+ - 2$  we assume a belief that the signal is L (this is pinned down by our belief monotonicity assumption when  $q_I^L = q_I^+ - 3$ ). The belief at  $Q = q_I^+$  is pinned down by our monotonicity assumption at a  $\frac{1}{3}$  probability of an H signal and  $\frac{2}{3}$  probability of an M signal. Finally, for all  $Q \geq q_I^+ + 2$  we assume a belief that the signal is H. Note that these assumed beliefs make downward deviations after M signals and upward deviations after H signals as unattractive as possible (these beliefs minimize the probability of acceptance following an M for downward deviations, and minimize potential trading profits following an H for upward deviations), and thus ensure existence of the equilibrium over the largest possible parameter space. As noted in the text, the most relevant potential deviations are upward deviations after an H signal and downward deviations after an M signal. From the text, a deviation to  $q_I^+ + 2$  after an H signal is profitable for all  $q_I^+ < 2$ , and with the minimum trade of  $q_I^+ = 2$  (which is the easiest such equilibrium to sustain), the blockholder will deviate down by two shares (which is shown below to be the most profitable deviation) after an M signal unless  $i \geq \frac{4(V_P - V_M)}{V_M - 1}$ .

For the active speculator case we formally specify out-of-equilibrium beliefs analogously to the no speculator case: the signal is believed to be L for all  $Q \leq q_I^+ - 3$  and H for all  $Q \geq q_I^+ + 3$ , while for  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , the monotone beliefs assumption requires the belief that the signal is H with probability  $\frac{1}{3}$  and M with probability  $\frac{2}{3}$ . As above, these beliefs ensure existence of the equilibrium over the largest possible parameter space. From the text, a deviation to  $q_I^+ + 2$  is profitable for all  $q_I^+ < 6$  following an H signal, and given the minimum trade of +6 the blockholder will deviate down by two shares after an M signal (which again is shown below to be the most profitable deviation) unless  $i \geq \frac{12(V_P - V_M)}{V_M - 1}$ .

The remaining issues not proven are: showing that the speculator's trades are incentive-compatible and individually rational in the active speculator case; and showing that the deviations considered above are the most relevant deviations. First consider the speculator's trades. Note that given the equilibria under consideration,  $S$ 's trade cannot affect  $D$ 's decision following any signal. Then denoting the expected value of the firm in equilibrium as  $E(V)$ ,  $S$ 's expected payoff is  $sE(V)$  no matter the quantity she trades since her trades are at zero expected profit or loss. To see this, note that the expected price of any of her trades is  $\frac{3}{4}V_P + \frac{1}{4}$ , while the expected value of the firm is  $\frac{1}{4}V_H + \frac{1}{2}V_M + \frac{1}{4}$ , which are equivalent (to see this, replace  $V_P$  with  $\frac{1}{3}V_H + \frac{2}{3}V_M$ ). Since a trade of zero is in the choice set, individual rationality is guaranteed.

We now show that we have focused on the relevant deviations for  $I$  in the text. First consider upward deviations after an H signal in the no speculator case. If  $I$  deviates up by three or more shares, the price is always  $V_H$ , so trading profits are eliminated. If  $I$  deviates up to  $q_I^+ + 1$ , the expected trading profit is  $\frac{1}{2}(q_I^+ + 1)(V_H - V_P)$ , which is lower than that derived for the 2-share deviation in the text. Next consider downward deviations after an M signal in the no speculator case. A downward deviation by three or more shares results in rejection by  $D$ , so the expected payoff is  $i$ . This is preferred to the equilibrium payoff if  $i > iV_M + q_I^+(V_M - V_P)$ , or, rearranging, if  $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$ , which is always harder to satisfy than the condition for the 2-share deviation in the text. A downward deviation by one share yields an expected payoff of

$i(\frac{1}{2}V_M + \frac{1}{2}) + \frac{1}{2}(q_I^+ - 1)(V_M - V_P)$  since  $D$  accepts half of the time, just as with the 2-share deviation. Since the trading quantity is higher, this expected payoff is clearly always lower than that for the 2-share deviation in the text.

For the active speculator case, consider upward deviations after an H signal. An upward deviation by one share has  $D$  still always accepting and yields an expected trading profit of  $\frac{3}{4}(q_I^+ + 1)(V_H - V_P)$ , which is clearly inferior to the 2-share deviation. A 3-share deviation again has  $D$  always accepting, and an expected trading profit of  $\frac{1}{4}(q_I^+ + 3)(V_H - V_P)$ , while a 4-share deviation has expected trading profit of  $\frac{1}{4}(q_I^+ + 4)(V_H - V_P)$ , which is clearly superior. The 2-share deviation profit is even higher if  $\frac{3}{4}(q_I^+ + 2) > \frac{1}{4}(q_I^+ + 4)$ , which always holds for  $q_I^+ > -1$  and thus always holds in the ranges where the equilibria may exist given the analysis in the text. Deviations up by more than four shares yield no trading profits.

Now consider deviations downward after an M signal in the active speculator case. Similar to the upward deviations, it is straightforward to show that a 2-share deviation is better than a 1-share deviation, and a 4-share deviation is better than a 3-share deviation (they have the same acceptance probability and lower trading losses). A 4-share deviation has a  $\frac{1}{4}$  probability of acceptance, leading to an expected payoff of  $i(\frac{1}{4}V_M + \frac{3}{4}) + \frac{1}{4}(q_I^+ - 4)(V_M - V_P)$ . Comparing this to the equilibrium payoff in the text, deviation is profitable if  $i < \frac{(q_I^+ + \frac{4}{3})(V_P - V_M)}{V_M - 1}$ , which is clearly harder to satisfy than the condition for the 2-share deviation in the text. A deviation by five or more shares has zero probability of acceptance, and thus expected payoff of  $i$ . This is preferred to the equilibrium payoff if  $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$ , which is again harder to satisfy than the 2-share deviation condition.

We must also show that  $I$  will not deviate either up or down after an L signal, and will not deviate *downward* after an H signal or *upward* after an M signal. With respect to the L signal, note that  $I$  makes no trading profit or loss in equilibrium (the price is always correctly one), and the value of his position  $i$  is maximized by non-acceptance since an acceptance is inefficient.

The only possibility for a trading profit with an L would be if  $I$  could *sell* some quantity for “too high” of a price and cause an inefficient acceptance some of the time (buying and having  $D$  accept is never optimal because he would be buying at too high of a price, leading to a trading loss). But this is impossible given the results above since a sale of one share would result in a maximum order flow of  $Q = 0$  in the no speculator case and  $Q = +1$  in the active speculator case, which is never sufficient for acceptance given  $q_I^+ \geq +2$  with no speculator and  $q_I^+ \geq +6$  with an active speculator. With respect to the H signal, note that deviating down will reduce the value of  $I$ 's initial position ( $D$  sometimes rejects) while also reducing his trading profits (there is no profit when  $D$  rejects). Similarly, after an M signal an upward deviation would leave the value of the initial position unchanged, but increase the trading loss since the price would sometimes be  $V_H$ . QED

**Proof of Proposition 2.** The proof proceeds by construction. First consider an equilibrium in the no speculator case in which  $q_I^H = +2$ ,  $q_I^M = 0$ , and  $q_I^L = -2$ . At order flow  $Q = +3$ , Bayes' Rule implies that the signal is H. At order flow  $Q = +1$ , the posterior is  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , so  $D$  accepts, which results in price  $V_P$ . We assume out-of-equilibrium beliefs are such that at order flow  $Q = 0$ , the signal is assumed to be M, that at all  $Q \geq +2$  the signal is assumed to be H, and that at all  $Q \leq -2$  the signal is assumed to be L. At all equilibrium order flows  $Q \leq 0$ ,  $\mu \leq \frac{1}{2}$  holds, so  $D$  rejects and the price equals one.

First note that deviations by  $I$  following an L signal are not optimal. The initial position  $i$  has its value maximized when  $D$  rejects (as always happens in equilibrium), and the only possibility of trading profits would arise if  $I$  could sell less and still have  $D$  sometimes accept. This is not possible since a sale of one share can never lead to acceptance (the maximum resulting order flow is zero). Next note that upward deviations by  $I$  after an H signal cannot be optimal. Any such deviation would have  $D$  always accepting, as in equilibrium, and would have trading profits of zero since the price would always be  $V_H$ .

Now consider downward deviations by  $I$  after an H signal. In equilibrium,  $D$  always accepts after an H, maximizing the value of  $i$ , and  $I$  has an expected trading profit of  $\frac{1}{2}(2)(V_H - V_P)$ . A deviation to  $+1$  means that  $D$  will accept only  $\frac{1}{2}$  of the time, and there are no trading profits (the trades are correctly priced at  $Q = 0$  and  $Q = +2$  given this deviation). A deviation to zero has no trading profits and  $D$  also accepts only  $\frac{1}{2}$  of the time, so this cannot be profitable. Similarly, upward deviations by  $I$  will not be profitable— $D$  always accepts at price  $V_H$ , so that trading profits are eliminated.

Finally,  $I$  has no incentive to deviate down after an M signal. In equilibrium,  $D$  accepts  $\frac{1}{2}$  of the time, and there are no trading profits/losses since he is not trading, i.e., the expected payoff is  $i(\frac{1}{2} + \frac{1}{2}V_M)$ . After a downward deviation,  $D$  will always reject and there are still no trading losses in equilibrium. Finally, consider an upward deviation after an M. A deviation up to  $+1$  cannot be optimal— $D$  still accepts  $\frac{1}{2}$  of the time, but now trading losses occur in those states. A deviation to  $+2$  has  $D$  always accepting—the expected payoff is  $iV_M - \frac{1}{2}(2)(V_P - V_M) - \frac{1}{2}(2)(V_H - V_M)$ . Comparing this to the equilibrium expected payoff shows that deviation is profitable if  $i > \frac{2(V_P - V_M) + 2(V_H - V_M)}{V_M - 1}$ , which equals  $2i^{*N}$ . Thus, this equilibrium exists for all  $i \in [0, 2i^{*N}]$ . QED

**Proof of Theorem 3.** The proof proceeds by construction. First assume  $\pi \in (\frac{2}{5} + \frac{2}{5}\lambda, \frac{1}{3} + \frac{2}{3}\lambda]$ . Consider an equilibrium in which  $q_I^M \geq +1$ ,  $q_I^H = q_I^M + 4$ , and  $q_I^L = q_I^M - 2$ . At the equilibrium order flows we have: if  $Q \in \{q_I^M + 4, q_I^M + 6\}$ ,  $D$  accepts and  $p(Q) = V_H$ ; if  $Q = q_I^M + 2$ ,  $D$  accepts and  $p(Q) = V_P$ ; if  $Q \leq q_I^M$ ,  $D$  rejects and  $p(Q) = 1$ . For out-of-equilibrium beliefs, we assume that for all  $Q \geq q_I^M + 3$  the signal is assumed to be H, for all  $Q \leq q_I^M - 3$  the signal is assumed to be L, for  $Q = q_I^M + 1$  the posterior is  $\mu = \pi$ , and at  $Q = q_I^M - 1$  the signal is assumed to be M or L with equal probability. For the out-of-equilibrium order flow  $Q = q_I^M + 1$ ,  $D$  is indifferent and is expected to accept with 50% probability. We label the corresponding value conditional upon acceptance as  $V_I = \pi(1 + d) + (1 - \pi)(1 - d + \epsilon)$ .



Now, consider  $S$ 's incentive to deviate.  $S$  enters the trading round with a position of magnitude  $s \geq 1$ . First assume this is a short position,  $-s$ . Then if the speculator short sells one share as the equilibrium requires, the possible equilibrium order flows are (from her perspective): if  $\theta = L$ ,  $Q \in \{q_I^M - 4, q_I^M - 2\}$  with equal probability (due to the noise trade); if  $\theta = M$ ,  $Q \in \{q_I^M - 2, q_I^M\}$  with equal probability; and if  $\theta = H$ ,  $Q \in \{q_I^M + 2, q_I^M + 4\}$  with equal probability. Thus,  $D$  will never accept after an M or L signal, and the price will always be one in those cases.  $D$  will always accept after an H and the price is  $V_H$  or  $V_P$  with equal probability. L, M, and H signals arrive with ex ante unconditional probabilities of  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$ , respectively. Thus, the expected price is  $\frac{3}{4} + \frac{1}{8}V_H + \frac{1}{8}V_P$ . The expected value of the shares is  $\frac{3}{4} + \frac{1}{4}V_H$ . The speculator's expected payoff to the equilibrium strategy is therefore  $-s(\frac{3}{4} + \frac{1}{4}V_H) - \frac{1}{8}(V_H - V_P)$  (trading losses occur only when  $D$  accepts at price  $V_P$  after an H), and the latter term represents the expected trading loss (which is easily shown to be equivalent to  $\frac{1}{4}(V_P - V_M)$  as stated in the result).

The only relevant deviation will be to not trade (buying will further reduce the value of  $s$  while also causing trading losses). With a deviation to zero, possible order flows are: if  $\theta = L$ ,  $Q \in \{q_I^M - 3, q_I^M - 1\}$ ; if  $\theta = M$ ,  $Q \in \{q_I^M - 1, q_I^M + 1\}$ ; and if  $\theta = H$ ,  $Q \in \{q_I^M + 3, q_I^M + 5\}$ . The only differences in outcomes are that  $D$  is now expected to accept after an M signal  $\frac{1}{4}$  of the time (noise buys  $\frac{1}{2}$  of the time, and then  $D$  is expected to accept  $\frac{1}{2}$  of the time when that happens). The speculator's expected payoff is therefore  $-s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$  since  $S$  is trading zero. Thus, selling yields an increase in the expected value of  $S$ 's stake of  $-s(\frac{3}{4} + \frac{1}{4}V_H) - (-s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)) = \frac{1}{8}s(V_M - 1)$  as given in the result. Comparing  $S$ 's expected payoff to the equilibrium payoff, the deviation is profitable if  $s < \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$ . Since we have assumed  $\frac{2(V_P - V_M)}{V_M - 1} < 1$  for this case,  $S$  will not deviate.

For the case with a long position of  $s$ , we similarly must check the deviation to no trade. Following similar logic, the equilibrium expected payoff to buying one share is  $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H) - \frac{1}{4}(V_P - V_M)$ . The expected payoff to not trading is  $s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$ . Comparing this

to the equilibrium payoff, the deviation is profitable if  $s < \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$ , so again  $S$  will not deviate. It is straightforward to show that  $S$ 's expected trading loss and the increase in the value of her position from buying rather than not trading are the same as in the case where  $S$  arrives short.

Next consider  $I$ 's possible deviations. There is no profitable deviation with an L since  $I$  cannot sell any quantity and have positive probability of acceptance.  $I$  will never optimally deviate upward with an H since all trading profits will be eliminated ( $D$  will always accept at price  $V_H$ ). Consider downward deviations after an H. It is straightforward to show that the most attractive deviation will be to  $q_I^M + 2$ . With this deviation,  $D$  either: accepts at  $Q = q_I^M + 4$  at price  $V_H$ , accepts at  $Q = q_I^M + 2$  at price  $V_P$ , or rejects at  $Q = q_I^M$ . The expected payoff is therefore  $i(\frac{1}{4} + \frac{3}{4}V_H) + \frac{1}{2}(q_I^M + 2)(V_H - V_P)$ . Comparing this to the equilibrium payoff, the deviation is profitable if  $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$ .

Next consider downward deviations after an M.  $I$ 's equilibrium expected payoff is  $i(\frac{3}{4} + \frac{1}{4}V_M) - \frac{1}{4}q_I^M(V_P - V_M)$ . If he deviates down by one share to  $q_I^M - 1$ ,  $D$  will accept with some probability only if  $Q = q_I^M + 1$ , and then with only  $\frac{1}{2}$  probability, which yields an expected payoff of  $i(\frac{7}{8} + \frac{1}{8}V_M) + \frac{1}{8}(q_I^M - 1)(V_M - V_I)$ . Comparing this to the equilibrium payoff, the deviation is profitable if  $i < \frac{(q_I^M + 1)(V_P - V_M) + (q_I^M - 1)(V_P - V_I)}{V_M - 1}$ . It is straightforward to show this is the most attractive deviation.

Now compare the 1-share downward deviation condition after an M *assuming*  $V_I = V_P$ ,  $i < \frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}$ , to the 2-share downward deviation condition after an H,  $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$ . By replacing the  $V$  terms with their algebraic definitions in terms of the model's primitives, it is straightforward to show that the former equals  $\frac{2(q_I^M + 1)Y}{3\epsilon}$  and that the latter equals  $\frac{q_I^M Y}{3\gamma}$ , where  $Y \equiv (2d - \epsilon)(2\lambda - \frac{1}{2})$  and  $\gamma \equiv (2d - \epsilon)\lambda + \frac{1}{2}(\epsilon - d)$ . Consider the ratio  $\frac{\gamma}{\epsilon}$ . Our assumption that  $V_L < 1$  implies  $\epsilon < \frac{d(4\lambda - 1)}{2\lambda}$ . We have  $\frac{\partial \gamma}{\partial \epsilon} = \frac{d(\frac{1}{2} - 2\lambda)}{\epsilon^2} < 0$ , and plugging for the maximum  $\epsilon$  we have  $\frac{\gamma}{\epsilon} = \frac{1}{2}$ , so  $\gamma \geq \frac{1}{2}\epsilon$  must always hold. Plugging this minimum  $\gamma$  into the expression for

the downward deviation condition following an H yields  $\frac{q_I^M Y}{3\frac{1}{2}\epsilon} = \frac{2q_I^M}{3\epsilon}$ , so the downward deviation condition following an M is always larger. Now note that the general condition for the 2-share downward deviation after an M,  $i < \frac{(q_I^M+1)(V_P-V_M)+(q_I^M-1)(V_P-V_I)}{V_M-1}$ , increases as  $V_I$  falls below  $V_P$ , and  $V_I \leq V_P$  always holds, so this is always the relevant downward cutoff for existence of the equilibrium.

Finally consider upward deviations after an M. It is straightforward to show that either the 1-share deviation to  $q_I^M + 1$  or the 2-share deviation to  $q_I^M + 2$  is the relevant deviation. The latter deviation yields an expected payoff of  $i(\frac{1}{4} + \frac{3}{4}V_M) - \frac{1}{4}(q_I^M + 2)(V_H - V_M) - \frac{1}{2}(q_I^M + 2)(V_P - V_M)$ . Comparing this to the equilibrium expected payoff, the deviation is profitable if  $i > \frac{(2q_I^M+5)(V_P-V_M)}{V_M-1}$ . The former deviation yields an expected payoff of  $i(\frac{1}{2} + \frac{1}{2}V_M) - \frac{1}{2}(q_I^M + 1)(\frac{1}{2}V_H + \frac{1}{2}V_I - V_M)$ . Comparing this to the equilibrium expected payoff, the deviation is profitable if  $i > \frac{(3q_I^M+4)(V_P-V_M)-(q_I^M+1)(V_P-V_I)}{V_M-1}$ . Thus, the relevant range of existence for this equilibrium is  $i \in [\frac{(q_I^M+1)(V_P-V_M)+(q_I^M-1)(V_P-V_I)}{V_M-1}, \min[\frac{(2q_I^M+5)(V_P-V_M)}{V_M-1}, \frac{(3q_I^M+4)(V_P-V_M)-(q_I^M+1)(V_P-V_I)}{V_M-1}]]$ .

Now note that at the minimum  $q_I^M$  we specified,  $q_I^M = +1$ , the lower boundary is clearly less than one given our assumption that  $\frac{2(V_P-V_M)}{V_M-1} < 1$ . Also, as  $q_I^M$  is increased, both the upper and lower boundaries of existence for the equilibrium increase, and the upper boundary is clearly always greater. Furthermore, the upper boundary clearly exceeds  $i^{*S}$  at a value of  $q_I^M = 5$ . Finally, note that the new lower boundary lies below the old upper boundary each time  $q_I^M$  is increased by one (plugging  $q_I^M + 1$  into the lower boundary yields  $\frac{(q_I^M+2)(V_P-V_M)+q_I^M(V_P-V_I)}{V_M-1} < \frac{(2q_I^M+2)(V_P-V_M)}{V_M-1}$ ), so considering each equilibrium as  $q_I^M$  increases by one from +1 to +5 yields the result.

Now assume  $\pi \in (\frac{1}{2}, \frac{2}{5} + \frac{2}{5}\lambda]$ . Consider an equilibrium in which  $q_I^M = +3$ ,  $q_I^H = +5$ , and  $q_I^L = -2$ . At the equilibrium order flows we have: if  $Q = +7$ ,  $D$  accepts and  $p(Q) = V_H$ ; if  $Q = +5$ ,  $D$  accepts and  $p(Q) = V_P^+$  where  $V_P^+ = \frac{1}{2}V_H + \frac{1}{2}V_M$ ; if  $Q = +3$ ,  $D$  accepts and  $p(Q) = V_P^-$  where  $V_P^- = \frac{1}{5}V_H + \frac{4}{5}V_M$ ; and if  $Q \leq +1$ ,  $D$  rejects and  $p(Q) = 1$ . For out-of-equilibrium beliefs, we assume that for all  $Q \geq +6$  the signal is assumed to be H, for all  $Q \leq 0$  the signal is assumed

to be L, for  $Q = +2$  the posterior is  $\mu = \pi$ , and at  $Q = +4$  the signal is assumed to be H with  $\frac{1}{3}$  probability and M with  $\frac{2}{3}$  probability (so that the relevant expected value is  $V_P$ ). For the out-of-equilibrium order flow  $Q = +2$ ,  $D$  is indifferent and is expected to accept with 50% probability.

Since the logic of the proof proceeds similarly to the above, for brevity we note here only the most relevant deviation possibilities. We first consider deviations by  $S$ . Assume  $S$  arrives short. As above, the only relevant deviation is not to trade.  $S$ 's equilibrium expected payoff upon selling one share can be written as  $-s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H) - \frac{9}{80}(V_H - V_M)$ . The latter term gives  $S$ 's expected trading loss, and is easily shown to be equivalent to  $\frac{27}{80}(V_P - V_M)$  as given in the result. If she deviates to not trading, her expected payoff is  $-s(\frac{3}{8} + \frac{3}{8}V_M + \frac{1}{4}V_H)$ . Comparing the two expected payoffs, deviation is profitable if  $s < \frac{\frac{27}{10}(V_P - V_M)}{V_M - 1}$ . It is straightforward to show that the problem is symmetric if  $S$  arrives long and the same trading loss and deviation condition applies. Thus, given our assumptions that  $\frac{\frac{27}{10}(V_P - V_M)}{V_M - 1} < 1$  for this case and  $s \geq 1$ ,  $S$  will not deviate. To see the expression for the gain in expected value given in the result, note that when  $S$  arrives short, she increases the value of her short position from  $-s(\frac{3}{8} + \frac{3}{8}V_M + \frac{1}{4}V_H)$  to  $-s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H)$  by selling short again, and symmetrically for the case when she arrives long.

It is also straightforward to show that the relevant remaining deviations that most tightly constrain the equilibrium's existence are deviations by  $I$  to trades of +1 or +5 after an M signal.  $I$ 's equilibrium expected payoff after an  $M$  signal can be expressed as  $i(\frac{1}{4} + \frac{3}{4}V_M) - \frac{27}{40}(V_H - V_M)$ . A deviation to +1 yields an expected payoff of  $i(\frac{3}{4} + \frac{1}{4}V_M) - \frac{1}{20}(V_H - V_M)$ . Comparing these, deviation is profitable if  $i < \frac{\frac{51}{20}(V_P - V_M)}{V_M - 1} < \frac{\frac{27}{10}(V_P - V_M)}{V_M - 1} < 1$ . A deviation to +5 yields an expected payoff of  $iV_M - \frac{55}{20}(V_H - V_M)$ . Comparing this to the equilibrium expected payoff, deviation is profitable if  $i > \frac{\frac{249}{10}(V_P - V_M)}{V_M - 1} > i^{*S}$ . QED

**Proof of Proposition 4.** First note that both thresholds are multiples of  $\frac{V_P - V_M}{V_M - 1}$ . Replacing the terms  $V_P$  and  $V_M$  with their expressions in terms of the model primitives yields  $\frac{1}{3} + \frac{\frac{2}{3}(4\lambda d - 2\lambda\epsilon - d)}{\epsilon}$ . Taking the derivative with respect to  $\epsilon$  yields  $-\frac{\frac{2}{3}d(4\lambda - 1)}{\epsilon^2} < 0$ . QED

**Proof of Lemma 2.** The proof is again by construction. First consider the full efficiency equilibrium analog to the no speculator case in which  $q_I^M = q_I^H = q_I^+ \geq +2$  and  $q_I^L = q_I^+ - 3$ . Now assume the speculator arrives long  $s$  shares (which is common knowledge) and is prescribed to buy one share. Possible equilibrium order flows are  $Q \in \{q_I^+, q_I^+ + 2\}$  following H and M signals depending on whether the noise trader buys or sells. Thus,  $D$  accepts at these order flows and the price is  $V_P$ . An L signal results in  $Q \in \{q_I^+ - 3, q_I^+ - 1\}$ , so  $D$  rejects for all  $Q \leq q_I^+ - 1$  and the price is one (out-of-equilibrium beliefs must place all weight on L in that range given monotone beliefs). Our monotone beliefs assumption also requires that at out-of-equilibrium node  $Q = q_I^+ + 1$ ,  $D$ 's posterior is  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , so he accepts and the price is  $V_P$ . Checking possible deviations by  $I$  proceeds as in the proof of Proposition 1 (note that  $I$  still cannot deviate to a “sell” quantity after an L and get  $D$  to accept, as selling one share leads to a maximum order flow of  $q_I^+ - 1$  if  $S$  buys, which is insufficient to get it accepted, and  $q_I^+ - 3$  if  $S$  is short and sells, which is again insufficient to get  $D$  to accept—see below), and it is straightforward to show that  $I$ 's incentive to deviate downward to  $q_I^+ - 2$  after an M again limits the range of existence to  $i \geq i^{*N}$ . The only remaining deviations to check are deviations by  $S$ .

In equilibrium,  $S$ 's expected payoff is  $s(\frac{1}{4} + \frac{1}{2}V_M + \frac{1}{4}V_H)$  ( $S$ 's trade is always executed at fair value from her perspective, so there is no expected trading profit or loss). If  $S$  deviates to zero, the M or H signal order flow becomes  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , so  $D$  will accept only  $\frac{1}{2}$  of the time, reducing the expected payoff to  $s(\frac{5}{8} + \frac{1}{4}V_M + \frac{1}{8}V_H)$ . If  $S$  deviates to  $-1$ ,  $D$  will again accept only  $\frac{1}{2}$  of the time, and there will again be no trading profit. Thus,  $S$  will not deviate. The proof for the case where  $S$  arrives short  $s$  shares is analogous.

Next consider the partial pooling equilibrium in which  $q_I^H = +2$ ,  $q_I^M = 0$ , and  $q_I^L = -2$ . Now assume the speculator arrives long  $s$  shares (which is common knowledge) and is prescribed to buy one share. The equilibrium order flow possibilities are: if the signal is H,  $Q \in \{+4, +2\}$ ; if the signal is M,  $Q \in \{+2, 0\}$ ; if the signal is L,  $Q \in \{-2, 0\}$ . Thus,  $D$  accepts at price  $V_H$  at  $Q = +4$ , accepts at price  $V_P$  at  $Q = +2$ , and rejects for lower  $Q$ . We assume out-of-equilibrium beliefs are such that  $D$  accepts at price  $V_H$  for all  $Q \geq +3$ , while  $D$  rejects for any  $Q \leq +1$  (the signal is believed to be M or L). Again, checking for deviations by  $I$  proceeds as in prior proofs and shows that the equilibrium exists for the entire range of  $i \in [0, i^{*N}]$ .

Finally, consider deviations by  $S$ .  $S$ 's equilibrium payoff is  $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H)$  (again, the trades are priced at fair value from her perspective). If  $S$  deviates to zero,  $D$  always rejects after an M and accepts  $\frac{1}{2}$  of the time after an H, so  $S$ 's expected payoff is  $s(\frac{7}{8} + \frac{1}{8}V_H)$ . A deviation to  $-1$  yields the same expected firm value, but there is a trading loss because  $D$  sometimes accepts after an H at price  $V_P$ , which is too low. The proof for when  $S$  arrives short is analogous. QED

**Proof of Theorem 5.** If  $i \geq i^{*S}$ , the full efficiency equilibrium will prevail in the final round, and all of the speculator's trades in both rounds will be correctly priced from her perspective (yielding zero expected profit/loss) and are therefore incentive-compatible.

Now assume  $i \in [1, i^{*S})$ . From Lemma 2 and the proceeding text we know that if the speculator and noise trader trade in the same direction in the first round, the speculator has zero profit/loss overall (both the first and final round trades occur at zero profit/loss). Note that all of the prices derived above for the active speculator case reflect a  $\frac{1}{2}$  probability that  $S$  will be long vs. short when her position entering the final trading round is unknown. Thus, using these prices it suffices to show that mixing with probability  $\frac{1}{2}$  between buying and selling one share in the first trading round is incentive-compatible and profitable for  $S$ . If the speculator does not trade, she gets an overall expected payoff of zero (it is easy to show she can never profit from trading against  $I$

with no initial position in the base model, so we assume she would not trade again, leading to an overall expected payoff of zero).

First we derive the first round market price assuming that the speculator's trade is not discovered (i.e., the noise trader trades in the opposite direction). Recall that in the range of  $i$  we need to consider, we have assumed the final round (base model) equilibrium is the relevant partial pooling equilibrium from Proposition 3. Now, given the order flow of zero, the market maker perceives that there is a 50/50 probability of  $S$  having gone long or short. If the speculator is short, the expected per share firm value is  $\frac{3}{4} + \frac{1}{4}V_H$ . If the speculator is long, the expected per share firm value is  $\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H$ . The overall expected firm value places  $\frac{1}{2}$  weight on each, which yields  $\frac{5}{8} + \frac{1}{4}V_H + \frac{1}{8}V_M$ , so this is the first round price when  $S$ 's trade is hidden.

$S$ 's expected payoff equals the sum of the expected trading profits (losses) from each round. These both equal zero if the first round trade is revealed (the noise trader trades in the same direction). Thus, the overall expected payoff to  $S$  if she buys one share can be expressed as  $\frac{1}{2} \left[ \left( \frac{1}{4}V_H + \frac{1}{4}V_M + \frac{1}{2} - \frac{5}{8} - \frac{1}{4}V_H - \frac{1}{8}V_M \right) - \frac{1}{4}(V_P - V_M) \right]$ , where the first term in the brackets is the expected first round trading profit, the second term is the expected final round trading loss, and the multiplication by  $\frac{1}{2}$  accounts for the probability that the trade is hidden by an offsetting noise trade (dropping the  $\frac{1}{2}$  yields the expression in the result, which is expected profit conditional on being hidden). This simplifies to

$$\frac{1}{2} \left[ \frac{1}{8}(V_M - 1) - \frac{1}{4}(V_P - V_M) \right]. \quad (8)$$

Similarly, the overall expected payoff if  $S$  sells one share can be expressed as  $\frac{1}{2} \left[ -\left( \frac{3}{4} + \frac{1}{4}V_H - \frac{5}{8} - \frac{1}{4}V_H - \frac{1}{8}V_M \right) - \frac{1}{8}(V_H - V_P) \right]$ , which simplifies to

$$\frac{1}{2} \left[ \frac{1}{8}(V_M - 1) - \frac{1}{8}(V_H - V_P) \right]. \quad (9)$$

Note that (8) and (9) are equivalent given  $V_P = \frac{1}{3}V_H + \frac{2}{3}V_M$ . This proves  $S$  is indifferent between buying and selling.

To prove the strategy is incentive-compatible, we must show that (8) and (9) are positive. From the proof of Proposition 4,  $\frac{2(V_P - V_M)}{V_M - 1} = 2 \left( \frac{1}{3} + \frac{\frac{2}{3}(4\lambda d - 2\lambda\epsilon - d)}{\epsilon} \right)$ . It is straightforward to show that our assumption that  $\frac{2(V_P - V_M)}{V_M - 1} < 1$  thus implies  $\epsilon > \frac{4d(4\lambda - 1)}{1 + 8\lambda}$ . Now note that  $\frac{1}{2} \left[ \frac{1}{8}(V_M - 1) - \frac{1}{8}(V_H - V_P) \right] > 0$  holds if  $(V_M - 1) - (V_H - V_P) > 0$ . Replacing the defined terms in this last expression with their definitions in terms of the primitives and rearranging yields  $d(\frac{2}{3} - \frac{8}{3}\lambda) + \epsilon(\frac{1}{6} + \frac{4}{3}\lambda)$ , which is positive if  $\epsilon > \frac{4d(4\lambda - 1)}{1 + 8\lambda}$ . QED

**Proof of Proposition 6.** Consider an equilibrium in which each blockholder is told to trade  $q_{Ij}^+$  after an H or M signal, and  $q_{Ij}^L$  such that  $q_{I1}^L + q_{I2}^L \leq q_{I1}^+ + q_{I2}^+ - 3$  after an L signal. Possible equilibrium order flows are as in the efficient equilibria for the single blockholder case with  $q_I^+$  replaced everywhere by  $q_{I1}^+ + q_{I2}^+$ , and  $q_I^L$  replaced by  $q_{I1}^L + q_{I2}^L$ . Out-of-equilibrium beliefs are also specified analogously (as discussed in the text with regard to the equilibrium in Fig. 2), which ensure existence of the equilibrium over the largest possible parameter space. As with the single blockholder case, it is easy to show that the most attractive deviations for each blockholder are to buy two more shares after an H signal, and buy two less after an M signal. With the former deviation, just as in the single blockholder case, either blockholder would lose trading profits with probability  $\frac{1}{4}$ , so that deviation is profitable if  $\frac{3}{4}(q_{Ij}^+ + 2)(V_H - V_P) > q_{Ij}^+(V_H - V_P) \implies q_{Ij}^+ < 6$ . Further, just as in the single blockholder model, if he deviates down to a trade of  $+4$  after an M signal, his deviation will be detected and the relationship will be rejected with only  $\frac{1}{4}$  probability, but he will save significant trading losses, which is profitable if  $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}4(V_M - V_P) > iV_M + 6(V_M - V_P) \implies i_j < \frac{12(V_P - V_M)}{V_M - 1}$ .

It remains to show that no other type of fully efficient pure strategy equilibrium can exist when  $i_1 = i_2 < \frac{12(V_P - V_M)}{V_M - 1}$ . Such equilibria can be asymmetric across the blockholders as long as  $q_{I1}^H + q_{I2}^H = q_{I1}^M + q_{I2}^M$ . First note that the results above imply that any such equilibrium must



have *each* blockholder trading at least six shares following an H signal (any less will result in a profitable deviation up by two shares). Thus, the minimum quantity of aggregate trade across the two blockholders is 12 shares after an M or H signal. Any efficient equilibrium must therefore require either that both blockholders trade at least six shares after an M, or that at least one trade more than six shares. But the results above imply that any blockholder required to trade six or more shares after an M will have an optimal deviation to a lower trade if their block is smaller than  $\frac{12(V_P - V_M)}{V_M - 1}$ . QED

**Proof of Proposition 7.** Directly calculating  $\frac{\partial i^{*N}}{\partial \lambda}$  yields  $\frac{8(2d - \epsilon)}{3\epsilon} > 0$ . Directly calculating  $\frac{\partial i^{*N}}{\partial d}$  yields  $\frac{8(2\lambda - \frac{1}{2})}{3\epsilon} > 0$ . Substituting  $\kappa d$  for  $\epsilon$  in  $i^{*N}$  yields  $\frac{8(\lambda(2 - \kappa) + \frac{1}{2}(\kappa - 1))}{3}$ , which is clearly independent of  $d$ . The proofs for  $i^{*S}$  are analogous. QED

**References**

- Admati, A., Pfleiderer, P., 2009. The Wall Street walk and shareholder activism: exit as a form of voice. *Review of Financial Studies* 22, 2645–2685.
- Allen, F., Gale, D., 1992. Stock price manipulation. *Review of Financial Studies* 5, 503–529.
- Asquith, P., Meulbroek, L., 1996. An empirical investigation of short interest. Unpublished working paper. Massachusetts Institute of Technology and Harvard University.
- Attari, M., Banerjee, S., Noe, T., 2006. Crushed by a rational stampede: strategic share dumping and shareholder insurrections. *Journal of Financial Economics* 79, 181–222.
- Bagnoli, M., Lipman, B., 1996. Stock price manipulation through takeover bids. *RAND Journal of Economics* 27, 124–147.
- Bakke, T., Whited, T., 2010. Which firms follow the market? An analysis of corporate investment decisions. *Review of Financial Studies* 23, 1941–1980.
- Bikhchandani, S., 1992. A bargaining model with incomplete information. *Review of Economic Studies* 59, 187–203.
- Boehmer, E., Jones, C., Zhang, X., 2009. Shackling short sellers: the 2008 shorting ban. Unpublished working paper. EDHEC Business School, Columbia University, and Purdue University.
- Bolton, P., von Thadden, E-L., 1998. Blocks, liquidity, and corporate control. *Journal of Finance* 53, 1–25.
- Burkart, M., Gromb, D., Panunzi, F., 1997. Large shareholders, monitoring, and the value of the firm. *Quarterly Journal of Economics* 112, 693–728.

Chakraborty, A., Yilmaz, B., 2004. Informed manipulation. *Journal of Economic Theory* 114, 132–152.

Chen, Q., Goldstein, I., Jiang, W., 2007. Price informativeness and investment sensitivity to stock price. *Review of Financial Studies* 20, 619–650.

Cohn, J., Rajan, U., 2012. Optimal corporate governance in the presence of an activist investor. Unpublished working paper. University of Texas and University of Michigan.

D'Avolio, G., 2002. The market for borrowing stock. *Journal of Financial Economics* 66, 271–306.

Diamond, D., Verrecchia, R., 1987. Constraints on short selling and asset price adjustment to private information. *Journal of Financial Economics* 18, 277–311.

Dow, J., Gorton, G., 1997. Stock market efficiency and economic efficiency: is there a connection? *Journal of Finance* 52, 1087–1129.

Drummond, B., 2006. Games short sellers play. *Bloomberg Markets*, September, 121–128.

Duffie, D., Garleanu, N., Pedersen, L., 2002. Securities lending, shorting, and pricing. *Journal of Financial Economics* 66, 307–339.

Durnev, A., Morck, R., Yeung, B., 2004. Value-enhancing capital budgeting and firm-specific stock return variation. *Journal of Finance* 59, 65–105.

Edmans, A., 2009. Blockholder trading, market efficiency, and managerial myopia. *Journal of Finance* 64, 2481–2513.

Edmans, A., 2011. Short-term termination without deterring long-term investment: a theory of debt and buyouts. *Journal of Financial Economics* 102, 81–101.

Edmans, A., Manso, G., 2011. Governance through trading and intervention: a theory of multiple blockholders. *Review of Financial Studies* 24, 2395–2428.

Edmans, A., Goldstein, I., Jiang, W., 2011. The real effects of financial markets: the impact of prices on takeovers. *Journal of Finance*, forthcoming.

Faure-Grimaud, A., Gromb, D., 2004. Public trading and private incentives. *Review of Financial Studies* 17, 985–1014.

Geczy, C., Musto, D., Reed, A., 2002. Stocks are special too: an analysis of the equity lending market. *Journal of Financial Economics* 66, 241–269.

Goldstein, I., Guembel, A., 2008. Manipulation and the allocational role of prices. *Review of Economic Studies* 75, 133–164.

Gul, F., Sonnenschein, H., 1988. On delay in bargaining with one-sided uncertainty. *Econometrica* 56, 601–611.

Holderness, C., 2009. The myth of diffuse ownership in the United States. *Review of Financial Studies* 22, 1377–1408.

Jarrow, R., 1992. Market manipulation, bubbles, corners, and short squeezes. *Journal of Financial and Quantitative Analysis* 27, 311–336.

Jones, C., Lamont, O., 2002. Short-sale constraints and stock returns. *Journal of Financial Economics* 66, 207–239.

Kahn, C., Winton, A., 1998. Ownership structure, speculation, and shareholder intervention. *Journal of Finance* 53, 99–129.

- Kalay, A., Pant, S., 2008. One share-one vote is unenforceable and sub-optimal. Unpublished working paper. University of Utah and University of Iowa.
- Kaplan, S., Moskowitz, T., Sensoy, B., 2010. The effects of stock lending on security prices: an experiment. Unpublished working paper. University of Chicago and Ohio State University.
- Khanna, N., Sonti, R., 2004. Value creating stock manipulation: feedback effect of stock prices on firm value. *Journal of Financial Markets* 7, 237–270.
- Khanna, N., Slezak, S., Bradley, M., 1994. Insider trading, outside search, and resource allocation: why firms and society may disagree on insider trading restrictions. *Review of Financial Studies* 7, 575–608.
- Kyle, A., 1985. Continuous auctions and insider trading. *Econometrica* 53, 1315–1336.
- Leland, H., 1992. Insider trading: should it be prohibited? *Journal of Political Economy* 100, 859–887.
- Luo, Y., 2005. Do insiders learn from outsiders? Evidence from mergers and acquisitions. *Journal of Finance* 60, 1951–1982.
- Maug, E., 1998. Large shareholders as monitors: is there a trade-off between liquidity and control? *Journal of Finance* 53, 65–98.
- Miller, E., 1977. Risk, uncertainty, and divergence of opinion. *Journal of Finance* 32, 1151–1168.
- Overstock.com, 2007. Complaint in superior court of the state of California, Case No. CGC-07-460147.

- Ozdenoren, E., Yuan, K., 2008. Feedback effects and asset prices. *Journal of Finance* 63, 1939–1975.
- Shleifer, A., Vishny, R., 1986. Large shareholders and corporate control. *Journal of Political Economy* 94, 461–488.
- Soros, G., 2009. *The Crash of 2008 and What it Means: The New Paradigm for Financial Markets*. PublicAffairs, New York.
- Subrahmanyam, A., Titman, S., 2001. Feedback from stock prices to cash flows. *Journal of Finance* 56, 2389–2413.
- Sunder, J., 2004. Information production in stock markets and cost of bank debt. Unpublished working paper. Northwestern University.
- Swinkels, J., 1999. Education signalling with preemptive offers. *Review of Economic Studies* 66, 949–970.
- Vila, J-L., 1989. Simple games of market manipulation. *Economics Letters* 29, 21–26.